A WIDE BAND ACCELERATION MEASURING, RECORDING AND ANALYZING SYSTEM

J. H. Arnold, Jr.
and
J. E. Colleary, Jr.

A WIDE BAND ACCELERATION MEASURING, RECORDING AND ANALYZING SYSTEM

by

J. H. ARNOLD, JR., Lieutenant, USN
B.A., Southwestern University, (1948)
B.S., U.S. Naval Postgraduate School, (1956)

J. E. COLLEARY, JR., Lieutenant, USN
B.S., U.S. Naval Academy, (1950)
B.S., U.S. Naval Postgraduate School, (1956)

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May, 1957

Thesis
A7257

This thesis, written by the authors while affiliated with the Instrumentation Laboratory, M.I.T., has been reproduced by the offset process using printer's ink in accordance with the following basic authorization received by Dr. C. S. Draper, Head of Department of Aeronautical Engineering and Director of the Instrumentation Laboratory.

COPY

March 1, 1956

Dr. C. S. Draper
Head of the Department of Aeronautical Engineering
and Director of the Instrumentation Laboratory

Dear Dr. Draper:

This is to authorize the deposit in the Library of permanent, offset-printed copies of theses published by the Instrumentation Laboratory in lieu of the ribbon copies normally required.

A WIDE BAND ACCELERATION MEASURING, RECORDING AND ANALYZING SYSTEM

by

J.H. Arnold, Jr.
J.E. Colleary, Jr.

Submitted to the Department of Aeronautical Engineering on May 20, 1957, in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

A proposal has been made for measuring, recording and analyzing the acceleration experienced by missile components and equipment through the use of commercially available equipment. Major emphasis is placed upon analyzing this acceleration in terms of its power spectral density. This is accomplished by breaking up the acceleration signal into a finite number of band widths by the use of band pass filters. The filter outputs are then measured by thermocouples which as square law devices give a direct indication of power spectral density. It is shown that power spectral density can be used as a guide to system design.

Thesis Supervisor: Walter Wrigley

Title: Professor of Aeronautical Engineering

36042

ACKNOWLEDGMENT

The authors express their appreciation to the personnel of the Instrumentation Laboratory , Massachusetts Institute of Technology, who assisted in the preparation of this thesis. Particular thanks are due to Kenneth Fertig, who, as immediate supervisor, inspired and guided the entire project, and to Robert Pinckney and Robert Martini for their interest and assistance in the practical aspects of the problem.

The graduate work for which this thesis is a partial requirement was performed while the authors were assigned to the U. S. Naval Administrative Unit, Massachusetts Institute of Technology.

TABLE OF CONTENTS

	Page
Object	11
Chapter 1 Introduction	13
Chapter 2 Method of Analysis	19
2.1 General	19
2.2 Randomness of Acceleration	19
2.3 Acceleration Power Density Spectrum	21
2.4 Power Spectral Density of a Time Function	23
2.5 Effect of Performance Function on Power Spectral Density	26
2.6 Effect of Band Pass Filters on Power Spectral Density	31
Chapter 3 Vibration Transducers	35
3.1 General	35
3.2 Accelerometer	36
3.3 Piezoelectric	38
3.4 Magnetic	40
3.5 Typical High Frequency Accelerometer	43
3.6 Typical Low Frequency Accelerometer	45
Chapter 4 Wide Band Acceleration Measuring System	55
4.1 General	55
4.2 High Frequency Branch	57
4.3 Low Frequency Branch	64
4.4 Summing Network and Amplifier	67
Chapter 5 Analyzing System	69
5.1 General	69
5.2 Recording Device	69

Page

5.3 Band Pass Filter 72

5.4 Thermocouple 75

Chapter 6 Conclusions and Additional Recommendations 81

6.1 General 81

6.2 Spectrum Analyzer 81

6.3 Extension of System 82

6.4 Impedance Reduction 82

6.5 Alternate Recording Method 82

Appendix A Output vs. Excitation Frequency Data of a Typical Low Frequency Accelerometer 85

Appendix B Filter Design 87

Appendix C Bibliography 95

LIST OF ILLUSTRATIONS

Figure		Page
1-1	Coherence of Acceleration Distribution Among Flights of Similar Missiles.	15
1-2	Wide Band Acceleration Measuring, Recording and Analyzing System.	17
2-1	Normal or Gaussian Distribution	20
2-2	Power Spectral Density vs. Frequency	22
2-3	Power Spectrum for Periodic Function.	24
2-4	Power Spectral Density of a Time Function vs. Frequency	28
2-5	Frequency Response of System	30
3-1	Pictorial Diagram of a Piezoelectric Accelerometer	41
3-2	Equivalent Circuit of a Piezoelectric Accelerometer	44
3-3	Output vs. Acceleration for the Typical High Frequency Accelerometer	46
3-4	Frequency Response for the Typical High Frequency Accelerometer	47
3-5	Variation in Sensitivity vs. Temperature for the Typical High Frequency Accelerometer	48
3-6	Output vs. Excitation Frequency for the Typical Low Frequency Accelerometer	50
3-7	Output vs. Acceleration for the Typical Low Frequency Accelerometer	51
3-8	Frequency Response for the Typical Low Frequency Accelerometer	52
3-9	Possible Normalized Frequency Response for Controlled Low and High Frequency Accelerometers	53

Figure		Page
4-1	Wide Band Acceleration Measuring System	56
4-2	High Frequency Branch of Measuring System	58
4-3	Desired Control of Frequency Response of High Frequency Branch	58
4-4	Basic Cathode Follower	59
4-5	Typical Cathode Follower Circuit Used with the Piezoelectric Accelerometer	61
4-6	Control of Frequency Response of Resistive-Capacitance Coupled Amplifier	63
4-7	Low Frequency Branch of Measuring System	65
4-8	Desired Control of Frequency Response of Low Frequency Branch	65
4-9	Control Desired by Low Pass Filter	67
4-10	Summing Network	68
5-1	Wide Band Analyzing System	70
5-2	Recorder Gain vs. Frequency	71
5-3	R-L-C Circuit and its Frequency Response	72
5-4	AC Equivalent Circuit	73
5-5	AC Filter Frequency Response	74
5-6	Frequency Response of Filter Network	76
5-7	Squared Current Input vs. Output Voltage for Thermocouple	77
5-8	Output vs. Frequency for Thermocouple	78
6-1	Demodulation and Filter Circuit for Alternate Recording Method	83
B-1	Effect of ζ on Gain vs. Frequency	93

OBJECT

The object of this thesis is to propose a system to measure, record and analyze, in terms of its power spectral density, the acceleration experienced by missile components and equipment.

CHAPTER I

INTRODUCTION

One of the major problems confronting missile and high speed aircraft component developers is the inadequacy of present systems to measure and reproduce some of the environmental conditions under which components and equipment function. The simulation of these environmental conditions is of importance if reliable missile components and equipment are to be developed.[1]*

The Skipper project at the Instrumentation Laboratory of Massachusetts Institute of Technology is greatly interested in environmental testing, and in systems that can simulate the environment under which a component functions during missile or aircraft flight. However, there are many questions concerning the design parameters of such a system that are as yet unanswered. It is believed that the best approach for obtaining these answers is through meeting realistic Military Specifications.

The importance of environmental test can be attested to by the emphasis placed on it by the Military. Environmental test by itself cannot assure reliability but without it reliability is not likely to be achieved. The Military has therefore compiled a number of requirements that components must meet in order that they be accepted as reliable.[2] These environmental test requirements are broken down into two main divisions. The first being non-operating requirements (storage life) and the second operating requirements (flight life); of prime interest to the

---

* Superscript numerals refer to similarly numbered references in Bibliography, Appendix C.

component developer is the division entitled operating requirements. As subheads under this requirement are many factors, but of major interest in developing a system for environment simulation is the subhead entitled vibration.

The situation example used for this thesis is the vibration existing in a missile. There are many sources of vibration existing in a missile during flight such as take-off forces, booster separation, electronic equipment and aerodynamic forces. With all these contributions, missile vibration appears to be made up of a wide distribution of frequencies with some predominating and can be stated as essentially a nonperiodic phenomenom.[3] However, to call it completely random is not quite correct, because there is a series of shocks and pulses occurring at normal intervals. So, it appears that missile vibration is composed of a predictable time profile upon which is superimposed a random function, (Fig. 1-1). Therefore, there is coherence in the distribution of acceleration in the frequency spectrum among flights of a class of missiles. This means that if a system is developed that will record the distribution of acceleration over one missile flight, in such a manner that it is able to be reproduced, this signal could be used for laboratory simulation of environmental conditions existing in all flights of similar missiles.[32]

Some effects of the quasi-random distribution of acceleration on electronic components are of great importance as far as reliability is concerned. Failures are likely to occur if coincidence is obtained between component resonances and airframe resonances. Insofar as possible this must be avoided.[2]

There are Military requirements set down for different situations pertaining to the acceleration under which an electronic component must function perfectly if it is to be accepted as reliable. One such requirement states that the component must function under an R.M.S. acceleration of 20 g's applied over a frequency range of 0 to 2000 cps. for 30 minutes in each of three

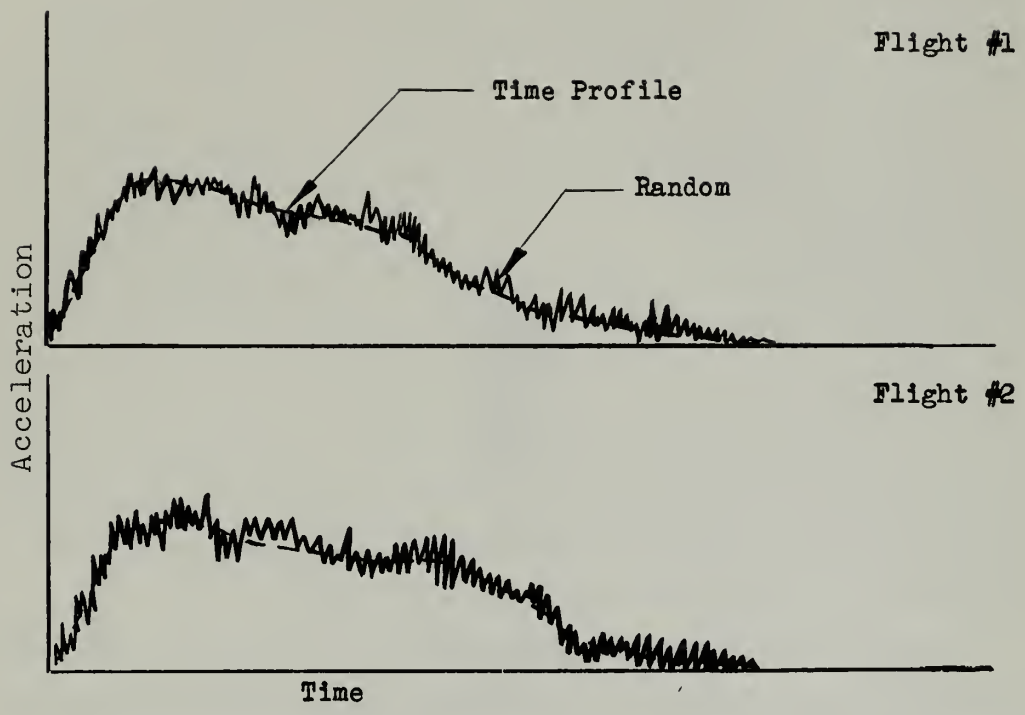


Fig. 1 - 1 Coherence of Aeceleration Distribution
Among Flights of Similar Missiles

mutually perpendicular directions. This gives an acceleration power spectral density of 0.2 $g's^2/cps$.[2]

Therefore, a system that is to record and reproduce this environment must function under the above specifications and must also be able to measure and record the above accelerations and analyze them in terms of their power spectral density. The proposed system uses these specifications as its basic design parameters.

At the present time acceleration measuring systems do not satisfactorily cover the continuous spectrum from 0 to 2000 cps. Some systems ignore the lower frequencies and sweep from 50 to 2000 cps while others ignore the higher frequencies and sweep from 0 to 70 cps. The proposed system will endeavor to correct this deficiency.

Consider the proposed system as made up of three parts, the measuring system, the telemetering or recording system, and the analyzing system, (Fig. 1-2). The system will only measure accelerations along one axis. Three such systems will have to be used to fully measure and reproduce actual accelerations.

For the measuring system it is believed that by mixing the outputs of two acceleration pick-ups, one a high natural frequency accelerometer and the other a low natural frequency accelerometer through suitable electronic amplifiers, a flat response can be obtained over the entire range from 0 to 2000 cps.

The combined output of the two bands can then be telemetered from the missile to an F.M. multichannel tape recorder system to setup a reproducible recording of wide bandwidth acceleration experienced by electronic components or equipment during missile flight. Similarly it could be directly recorded in a piloted aircraft.

This recorded signal, or the original combined output signal from the measuring system, can then be used to drive a wide band current amplifier. The output of this amplifier is used in two ways. First, the amplifier output is used to drive a single thermocouple to obtain the total mean square acceleration. Then, the output is used as an input to a series of narrow band pass filters whose individual outputs are terminated in thermocouples. In this way it is believed that a line spectral approximation of the acceleration power spectral density can be obtained.

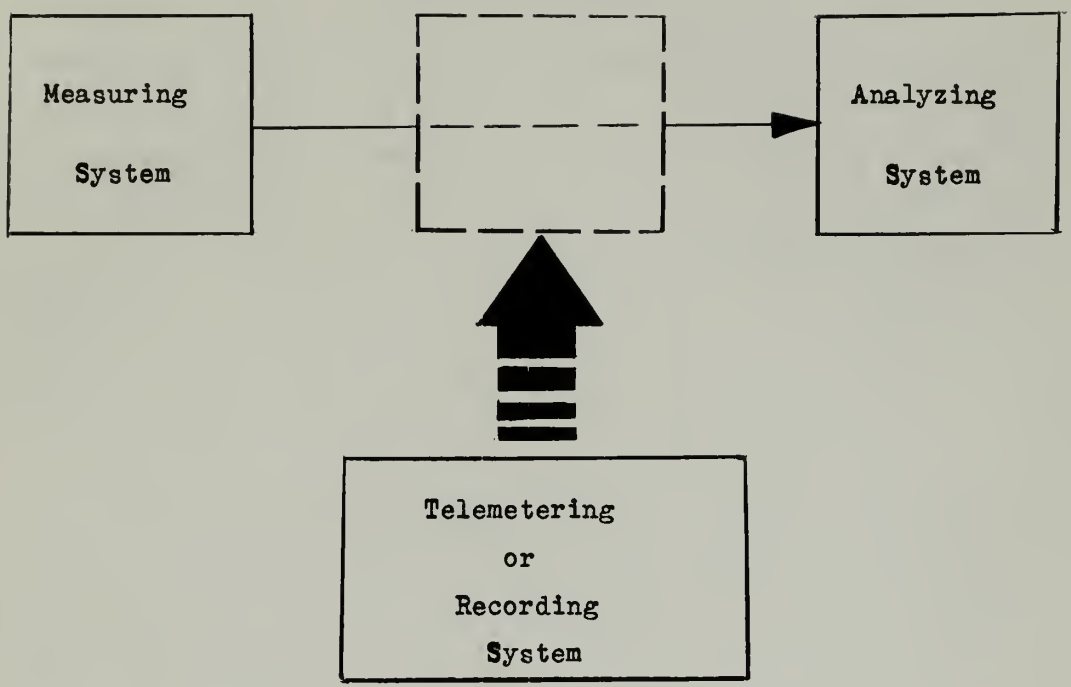


Fig. 1 - 2 Wide Band Acceleration Measuring, Recording and Analyzing System.

Furthermore, these thermocouple outputs can be amplified and recorded to provide a time history of a slowly varying acceleration power spectrum such as commonly found in maneuvering aircraft or missiles in flight.

This spectrum of acceleration is not only of use in complying with Military Specifications but may also be used to approximate correlation functions, whose direct determination through instrumentation might be difficult, in order to relate the input-output characteristics of a structure or a component to wide band acceleration.

CHAPTER 2

METHOD OF ANALYSIS

2.1 General

It is shown in this chapter that random accelerations are predictable in a statistical sense. The statistical measure is the power spectral density which is defined. It is shown that random functions have a power density spectrum and that the analysis can be applied to acceleration measurements.

A system's output depends not only on its input but also on its Performance Function. It is shown that the Performance Function affects the Power Spectral Density input and how this Performance Function may be estimated by measurements of its input and output Power Spectral Density.

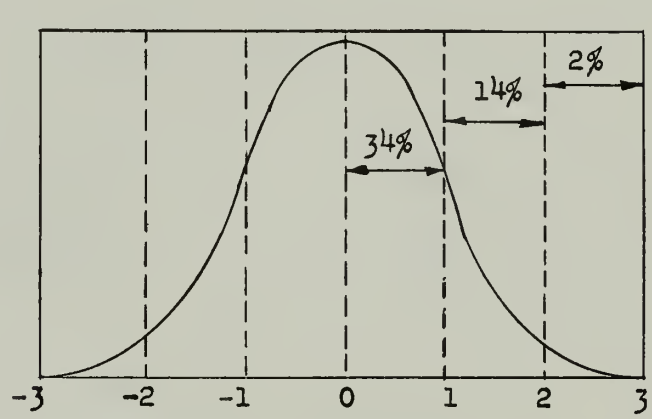
An ideal filter will pass all of the energy present within its bandwidth. It is shown that the energy can be represented by the integral of the power spectral density over the bandwidth. With other than ideal filters it is shown how the frequency response affects the energy output. The summed outputs of the actual filters give a line spectral approximation to the acceleration density spectrum.

2.2 Randomness of Acceleration

Component failure due to imposed accelerations is of prime importance in missile design. It has been found that a large percentage of failures are due to random accelerations; accelerations which do not just happen but appear to follow a

statistical plan. They are the result of a finite number of effects acting independently and mostly nonperiodic. The sum result is an acceleration which has certain predictable characteristics.[4]

In Fig. 2-1 a probability plot is presented which shows the possibility of certain magnitude ratios occurring.[3] During a period of time, 68% of the instantaneous acceleration will be between plus and minus one, about 5% will exceed plus and minus two, and about 0.3% will exceed plus and minus 3.



$$AR = \frac{\text{instantaneous acceleration}}{\text{rms acceleration}}$$

Fig. 2 - 1 Normal or Gaussian Distribution

Since the specified rms acceleration is 20 g's,[2] peak accelerations of 60 g's and greater should be expected approximately 0.3% of the time period. It is for this reason that the accelerometers and associated devices should be able to withstand at least 60 g's.

2.3 Acceleration Power Density Spectrum

A basic yardstick for component design acceptance tests is the "mean square acceleration per cycle per second bandwidth"[3] This introduces a measure for comparison purposes of random acceleration. This measure is called "acceleration density" and is defined as

$$\text{Acceleration Density} = \lim_{BW \to 0} \frac{g_{rms}^2}{BW} \qquad [2\text{-}1]$$

where g_{rms} is the root mean square of the average accelerations in the bandwidth(BW). Mean Square is g_{rms}^2 and a measure of the power present at frequencies within the bandwidth.

Acceleration density has been given other names but in this application it is generally called Power Spectral Density (PSD).

A graph showing MS acceleration vs. frequency would show the effect of random acceleration at the frequencies of interest during any period of missile flight. Random means the accelerations would not be exactly reproduced during a subsequent run, however, the Mean Square would be approximately the same for any identical period of time during a subsequent missile run.

The Military specification called for an acceleration of 20 $g's_{rms}$ or less over the frequency range 0-2000 cps.[2] In terms of PSD this is

$$\frac{(20)^2}{(2000-0)} = 0.2 \ g^2/\text{cps for a flat PSD input.}$$

(Fig. 2-2).

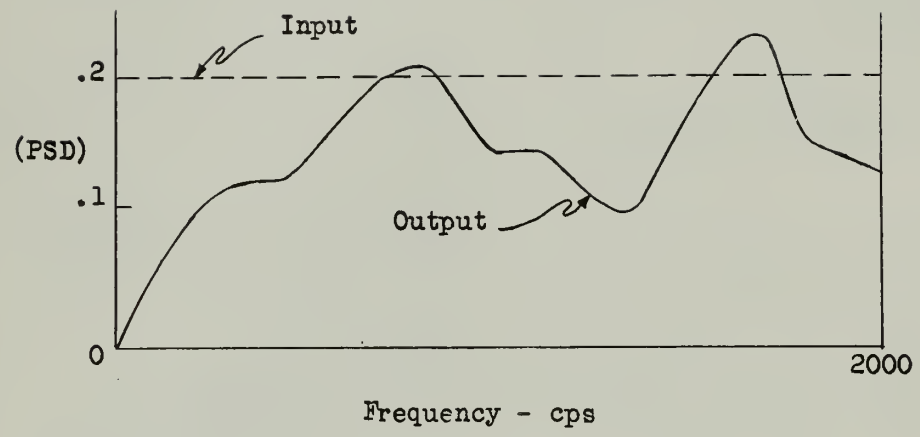


Fig. 2 - 2 Power Spectral Density vs. Frequency

The dashed line indicates the flat PSD input to a system during a time interval Δt. The output curve indicates the measured PSD output of the system. It is also a plot of transmissibility versus frequency for the system. This is an indication of the amplitude ratio portion of the system Performance Function.

It can be seen that peaks exceed the specification and are close at other points. Components should be constructed so that their natural frequencies occur away from these points so that damage is least likely to be expected.

2.4 Power Spectral Density of a Time Function

The different effects which determine the accelerations experienced by the missile are not constant for the most part during the entire run. However, they are, as functions of time, similar from run to run.[3] i.e.,

1) burning fuel effects
2) resulting shift in c.g.
3) separation of booster phases
4) forces caused by guidance and control mechanisms
5) take-off
6) density change effects
7) air turbulence, pressure variations
8) torsion due to steering and stabilizing.

Therefore, it is assumed that the recorded accelerations obtained during the period of a flight approximate accelerations experienced on any similar, subsequent flight.

It is also assumed that the run will be broken into small enough time intervals so that the random accelerations can be considered stationary during each time interval. Using this last assumption the following proofs are presented for these cases of wave forms.[5][6]

a. Periodic Function

A periodic function can be broken into a Fourier Series which expresses the function as a summation of sinusoids

$$f(t) = \sum_{0}^{\infty} (a_n \cos \omega_n t + b_n \sin \omega_n t) \qquad [2\text{-}2]$$

Power occurs at discrete frequencies $\omega_n = \frac{2\pi n}{T}$ and at each frequency equals $(a_n^2 + b_n^2)$.

$$\text{The total power} = \lim_{T \to \infty} \int_{-T}^{T} f^2(t)dt = \sum (a_n^2 + b_n^2) \qquad [2\text{-}3]$$

$$\text{Average power} = \frac{\sum (a_n^2 + b_n^2)}{2T} \qquad [2\text{-}4]$$

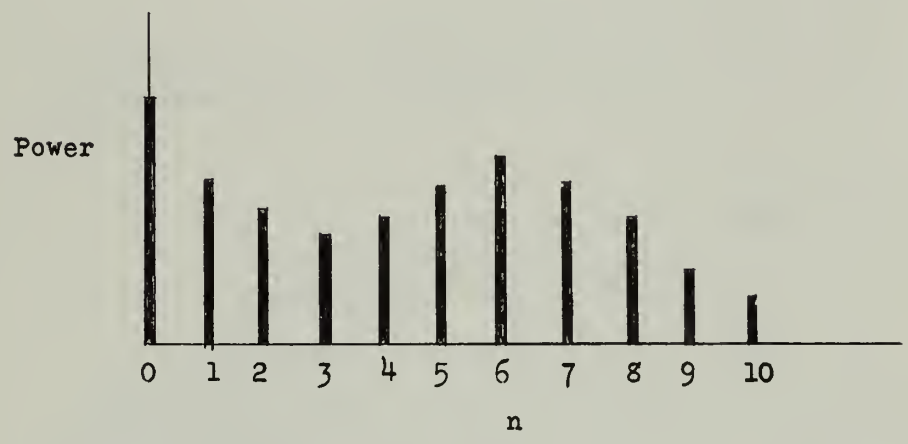


Fig. 2 - 3 Power Spectrum for Periodic Function

b. Nonperiodic function

A nonperiodic function that is zero at $\pm\infty$ cannot be broken into a Fourier Series but it can be expressed as a sum of sinusoids by the Fourier Transform, $F(\omega)$.

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt \qquad [2\text{-}5]$$

then

$$f(t) = \int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, d\omega \qquad [2\text{-}6]$$

Power, $|F(\omega)|^2$ occurs continuously over all frequencies and total power is

$$\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t)^2 \, dt \qquad [2\text{-}7]$$

$|F(\omega)|^2$ is defined as the "power spectral density" of f(t)

c. Random Function

Consider now a nonperiodic function which is finite at $\pm\infty$. (e.g. random accelerations). The integral $\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$ blows up so the Fourier Transform does not work. It cannot be expressed as a Fourier series.

However, a $\Phi(\omega)$ corresponding to $|F(\omega)|^2$ or $(a_n^2 + b_n^2)$ can be gotten and this turns out to be the Fourier transform of $\phi(\tau)$, the correlation function.

$$\phi(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t) \, f(t+\tau) dt \qquad [2\text{-}8]$$

and

$$\Phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(\tau) \, e^{-i\omega\tau} \, d\tau \qquad [2\text{-}9]$$

$$\text{Average Power} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f^2(t) \, dt = \phi(0) \qquad [2\text{-}10]$$

$$\text{and} \qquad \phi(0) = \int_{-\infty}^{\infty} \Phi(\omega) e^{i\omega\tau} \, d\omega = \int_{-\infty}^{\infty} \Phi(\omega) d\omega \qquad [2\text{-}11]$$

Again, $\Phi(\omega)$ is the "power spectral density" of f(t).

It can be shown that correlation function analysis applies to all three cases, and results in repeated spikes for the periodic case which show up as lines of finite area in a Power Spectral Density function.

Thus it can be seen that random acceleration, if considered stationary, can be expressed in terms of power spectral density.

2.5 Effect of Performance Function on Power Spectral Density[7]

In this section it is shown, by methods similar to those used in Article 2.4, that the Performance Function of a system subject to an input operates on this input to produce a new Power Spectral Density function. It is further shown that if the input and output functions are known an estimate of the Performance Function can be made.

Take the function f(t) previously discussed in Article 2.4. The average f is

$$f_{av} = \frac{1}{n} \sum_{i=1}^{n} f_i \qquad [2\text{-}12]$$

where f_i is the value of f(t) sampled n times.

The Mean Square of f(t) is obtained by a similar method.

$$[MS]_f = \frac{1}{n} \sum_{i=1}^{n} f_i^2 \qquad [2\text{-}13]$$

The Correlation Function of a time function is defined as the time average.

$$(CF)_f = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t)\, f(t+\tau)\, dt \qquad [2\text{-}14]$$

and it can be seen that for $\tau = 0$

$$[MS]_f = \left[(CF)_f\right]_{\tau=0} \qquad [2\text{-}15]$$

But $(CF)_f$ is defined in the time domain and in order to derive a frequency analysis it is necessary to define the Fourier Transform of the correlation function as

$$(PSD)_f = [\text{Fourier Transform}]\,[(\text{Correlation Function})_f] \quad [2\text{-}16]$$

from this:

$$(PSD)_f = \frac{1}{\pi}\int_{-\infty}^{\infty} [(CF)_f]\, e^{-i\omega\tau}\, d\tau \quad [2\text{-}17]$$

and $(PSD)_f$ is the Power Spectral Density of f(t).

Since $(CF)_f$ is an even function of τ the following can be said:

$$(PSD)_f = \frac{1}{\pi}\int_{-\infty}^{\infty} [(CF)_f] \cos\omega\tau\, d\tau \quad [2\text{-}18]$$

which is the same as

$$(PSD)_f = \frac{2}{\pi}\int_{0}^{\infty} [(CF)_f] \cos\omega\tau\, d\tau \quad [2\text{-}19]$$

from eq. [2-16]

$$(CF)_f = [FT]^{-1}\ [(PSD)_f] \quad [2\text{-}20]$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} [(PSD)_f]\ e^{i\omega\tau}\, d\omega \quad [2\text{-}21]$$

again as in [2-18]

$$(CF)_f = \int_0^\infty [(PSD)_f] \cos \omega \tau \, d\omega \qquad [2\text{-}22]$$

From [2-15] when $\tau = 0$

$$[MS]_f = \int_0^\infty [(PSD)_f] \, d\omega \qquad [2\text{-}23]$$

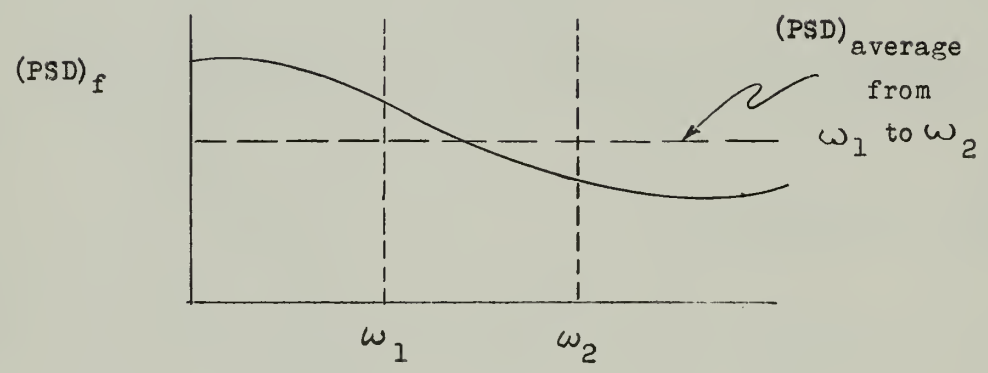


Fig. 2 - 4 Power Spectral Density of a Time Function vs. Frequency

From Fig. 2-4 it can be seen that from ω_1 to ω_2

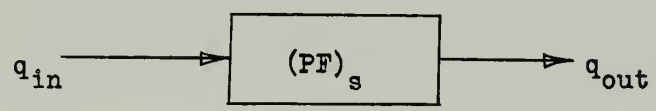
$$[(MS)_f]_{\Delta\omega} = [(PSD)_f]_{AV.} [(\Delta\omega)] \qquad [2\text{-}24]$$

Transposing:

$$\left[(PSD)_f\right]_{AV.} = \frac{\left[(MS)_f\right]_{\Delta\omega}}{\left[(\Delta\omega)\right]} \qquad [2\text{-}25]$$

This means that the average power spectral density over a band of frequencies is the total Mean Square in the bandwidth divided by the length of the bandwidth.

Using the preceeding results, the effect of Performance Function on $(PSD)_f$ is now obtained:



$$q_{out} = (PF)_s \quad (q_{in}) \qquad [2\text{-}26]$$

Transposing:

$$\frac{q_{out}}{q_{in}} = (PF)_s \qquad [2\text{-}27]$$

but

$$(PF)_s = (AR)_s \quad e^{j(PA)_s} \qquad [2\text{-}28]$$

where $(AR)_s$ is Amplitude Ratio
and $(PA)_s$ is the Phase Angle
} as a Function of ω

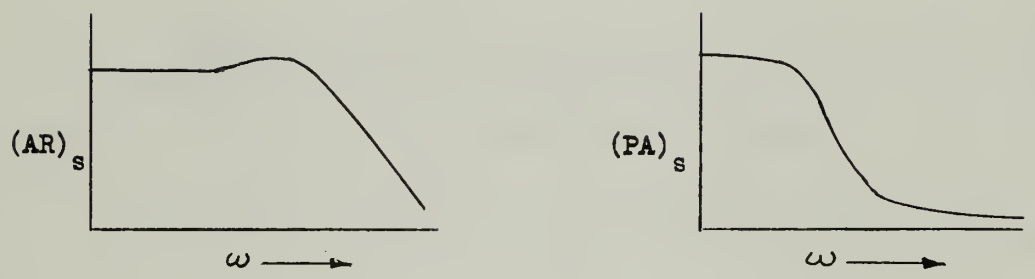


Fig. 2 - 5 Frequency Response of System

From eqs. [2-26] and [2-13]

$$(MS)_{q_{out}} = (PF)_s^2 \; (MS)_{q_{in}} \qquad [2\text{-}29]$$

substituting [2-28]

$$(MS)_{q_{out}} = \left| AR_s \right|^2 \; (MS)_{q_{in}} \qquad [2\text{-}30]$$

from [2-25]

$$(PSD)_{q_{in}} = \frac{(MS)_{q_{in}}}{\Delta\omega} \quad \text{and} \quad (PSD)_{q_{out}} = \frac{(MS)_{q_{out}}}{\Delta\omega}$$

and from [2-30]

$$(MS)_{q_{out}} = \left|AR_s\right|^2 (PSD)_{q_{in}} (\Delta\omega) \qquad [2\text{-}31]$$

substituting again from [2-25]

$$(PSD)_{q_{out}} = \left|AR_s\right|^2 (PSD)_{q_{in}} \qquad [2\text{-}32]$$

Transposing

$$\frac{(PSD)_{out}}{(PSD)_{in}} = \left|AR_s\right|^2 \qquad [2\text{-}33]$$

Thus, at any fixed ω, measuring $(PSD)_{in}$ and $(PSD)_{out}$ will give the $\left|(AR)_s\right|^2$ at that ω. Repeating this and varying ω from zero to infinity will result in the amplitude ratio portion of the system Performance Function. We are not interested in the phase angle since phase information is lost in the thermocouple measurement.

If AR_s were that from a linear system its phase and time response are unique.

2.6 Effect of Filters on Power Spectral Density

a) Ideal Filter(5)

Let f(t) be the input to an ideal bandpass filter whose frequency response is $A(j\omega)$ with:

$$\left|A(j\omega)\right| = \begin{cases} 1 & \omega_1 \leq \omega \leq \omega_2 \\ 0 & \omega_2 < \omega < \omega_1 \end{cases} \qquad [2\text{-}34]$$

Say $g(t)$ is the output of the filter and let its transform be $G(\omega)$

$$G(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(t) e^{-i\omega t} \, dt \qquad [2\text{-}35]$$

$$\text{also} \qquad G(\omega) = A(j\omega) \; F(\omega) \qquad [2\text{-}36]$$

The total energy output of the filter is that part of the energy of $f(t)$ lying in the frequency band ω_1 to ω_2 and is given by

$$\int_{-\infty}^{\infty} g^2(t) \, dt = \int_{-\infty}^{\infty} |G(\omega)|^2 \, d\omega \qquad [2\text{-}37]$$

substituting from [2-36]

$$\int_{-\infty}^{\infty} g^2(t) \, dt = \int_{-\infty}^{\infty} |A(j\omega)|^2 \, |F(\omega)|^2 \, d\omega \qquad [2\text{-}38]$$

$$\text{Since } |A(j\omega)|^2 = 1 \text{ or } 0 \qquad [2\text{-}39]$$

$$\int_{-\infty}^{\infty} g^2(t) \, dt = \int_{\omega_1}^{\omega_2} |F(\omega)|^2 \, d\omega \qquad [2\text{-}40]$$

thus the energy of $f(t)$ lying in the band ω_1 to ω_2 is given by the integral of PSD over ω_1 to ω_2 and the total energy of

$$f(t) = \int_{0}^{\infty} |F(\omega)|^2 \, d\omega . \qquad [2\text{-}41]$$

Therefore, the mean square acceleration (or energy) of f(t) lying in the frequency band 0-2KC is equal to the summation of mean squares as measured by the ideal bandpass filters which fully cover this bandwidth.

b) Actual Filter

The filter designed in Article 5.3 does not have the frequency response of the ideal filter, Equation [2-12]. Its response is plotted in Fig. B 1 and it can be seen that the response falls off on either side of the center frequency. Thus Equation [2-16] becomes

$$\int_{-\infty}^{\infty} g^2(t)dt = \int_{-\infty}^{\infty} \left|AR\right|^2 \left|F(\omega)\right|^2 d\omega \qquad [2\text{-}42]$$

Bandwidth filters with appropriate characteristics can be properly overlapped so that the Mean Square approximation is a sufficient representation of the energy present.

$$\text{Mean Square Acceleration} \Big]_0^{2KC} = \sum_{i=1}^{n} \int_0^{2KC} \left|(AR)_i\right|^2 \left|F(\omega)_i\right|^2 d\omega \qquad [2\text{-}43]$$

In subsequent chapters it will be shown how and in what terms acceleration is measured so that values for [2-42] and [2-43] can be computed. [2-42] is the Mean Square acceleration within the bandwidth ω_1 to ω_2 and when divided by the bandwidth is the Power Spectral Density for the bandwidth.

CHAPTER 3

VIBRATION TRANSDUCERS

3.1 General

When it becomes necessary to measure complex and severe vibrations a transducer of some type should be employed. For present day measurement of vibrations on typical miniature components a very small, accurate and rugged transducer is desired. A large number of specialized transducers are available for use under particular operating environment. In order to measure the frequency and magnitude of vibration in a particular device the engineer must select the transducer that is best suited for his job.

This selection in general should be guided by the following parameters:[8]

1) the type of input to be measured;
2) the frequency range of this input;
3) the amount of space available for mounting;
4) the allowable weight, in order that the transducer will not affect appreciably the output of the device to be measured;
5) the operating temperature range;
6) the dynamic performance;
7) the availability;
8) the cost and
9) its compatibility with the rest of the system.

Considering the above selection parameters with relation to this system, a self contained electrical signal generating transducer seems to be the optimum. This type of transducer converts either directly or indirectly the vibration energy into an equivalent electrical energy. This electrical energy is especially useful when continuous records of vibration are required because of its direct use with commonly available electrical recording equipment.

With this type of transducer in mind the field can be further narrowed down by the type of measurement that is desired:

1) vibratory displacement;

2) vibratory velocity or

3) vibratory acceleration

Normally vibratory displacement and vibratory velocity transducers can be ruled out for measuring complex missile component vibrations. The self contained vibration displacement instrument would be excessively large and would have frequency limitations in the operating range. The velocimeter would have both high and low frequency limitations.

Therefore, where complex missile component vibrations are to be measured, it is believed that acceleration measurement is the most advantageous. This is also borne out by the requirements set forth in the Military Environmental Test Requirements, as stated in Chapter 1. Thus a self contained electrical signal generating acceleration type transducer called an accelerometer was selected as the pick-up for the measuring system.

3.2 Accelerometer

An accelerometer is a linear, single-degree-of-freedom electro-mechanical device that when used below its natural

frequency generates a voltage proportional to accelerations to which it has been subjected. The differential equations of motion and characteristics are treated in many standard and classical texts and will not be discussed in this thesis.[8][9]

The best type of accelerometer must be selected from the many types available. The choice of the particular type again depends on the selection parameters noted earlier in the chapter.

It is desired to have an accelerometer that will accept a maximum acceleration of 60 g's or greater with a frequency range of 0 to 2000 cps and whose frequency response is flat over this range. The maximum acceleration figure was selected because statistics indicate that only a small number of acceleration peaks will exceed this figure. This has been discussed in Chapter 2, Article 2.2. The frequency range requires an accelerometer with a natural frequency of around 4000 cps or greater. This natural frequency is easily accomplished with commercially available accelerometers at the cost of an attenuated output signal at low frequencies.[10] Therefore, it will be necessary to improve the low frequency response of the wide band accelerometer. To accomplish this it has been decided to employ a second accelerometer of low natural frequency. The outputs of the two accelerometers are combined in such a manner as to give a flat frequency response over the desired range.

The possibility that the high frequency limitation of 2000 cps might be raised for future studies has also influenced the selection of the dual accelerometer measuring system.

Considering now the many types of commercially available accelerometers in the light of the selection parameters and the desired performance discussed above, the optimum accelerometer combination will be selected.

The resistive or the potentiometer accelerometer is useful at low frequencies but is frequently large and requires a fairly large input energy. Its output sensitivity is high, but the signal is noisy due to stiction effects. Thus it was ruled out as a possible low frequency accelerometer. Many of the other types of accelerometers may also be ruled out because of their size and frequency limitations. However, it does appear that two types of accelerometers meet the selection parameters. They are the piezoelectric and magnetic types.

3.3 Piezoelectric

The piezoelectric principle is particularly suited for accelerometers in component testing of guided missiles.[33]

The principle is that effect which exists when an asymmetrical crystalline material, acted upon by a mechanical force or stress, developes an electric potential upon certain surfaces through the changing of its dimensions. A crystal that has this property is called a piezoelectric crystal.[12] When the crystal is employed in an accelerometer the accelerometer is very directional in nature; the sensitive direction is called the input axis. It is almost insensitive to acceleration applied at angles perpendicular to the input axis. The amount of sensibility to these perpendicular accelerations is called crosstalk.

The most common piezoelectric crystals are quartz, which occurs in a natural state, Rochelle salt and barium titanite, which are synthetic.[12]

The natural crystals have the advantage of very low leakage which when used with a very high impedance input amplifier, permits the measurement of slowly varying forces. They are capable of operating with reasonably low frequency variations as well as withstanding high temperature and more rugged applications,

such as shock.[12]

One of the difficulties in the use of natural piezoelectric crystals as transducers is that the outputs are relatively small in magnitudes for most applications and this requires the use of amplifiers and other circuitry including connecting cables which are capable of introducing errors.

Errors are produced by the capacitance of the connecting cables in the input circuit of the amplifier, and it is generally desired to place the amplifiers as close to the piezoelectric transducer as possible since shorter leads will usually improve the response characteristics.[11]

Synthetic crystals, on the other hand, have a marked advantage of greater output sensitivity, higher output for a given force. They are also able to be constructed in almost any given size or shape. But they are usually unable to withstand high mechanical strain directly without fracture, and they deteriorate more rapidly at high temperatures than natural crystals.

Rochelle salt has a large piezoelectric effect so that these accelerometers have, as a rule, high sensitivity. Another great advantage of Rochelle salt is that it is possible to construct the crystal elements with a relatively small internal impedance, so that the accelerometer can stand being loaded by the capacitance of a cable between the accelerometer and a subsequent amplifier, and the input impedance of the amplifier need not be extraordinarily high in order to obtain frequency linearity even at the lower frequencies.

Properly prepared and polarized, barrium titanite also combines a uniformly high piezoelectric effect with large capacity and electrical stability as well as good mechanical strength. Its strength, when exposed to pressure, can be used

when constructing accelerometers by allowing the weight to press on a relatively small piezoelectric crystal, so that this crystal is exposed to pressure variations when experiencing accelerations.[11] (Fig. 3-1)

Piezoelectric elements are also somewhat less subject to change in sensitivity from aging than other types of primary elements and need only infrequent recalibration.[12]

Therefore, in selecting a high frequency accelerometer for this wide band acceleration measuring system a synthetic crystal piezoelectric accelerometer was chosen. This accelerometer will accept a high maximum acceleration, has an excellent high frequency response and is small and light weight. Typical commercially available accelerometers of this type are manufactured by the Endevco Corporation[15][16] and the Gulton Manufacturing Corporation.[17][18]

A low frequency accelerometer must now be chosen from the magnetic group that is compatible with the selected high frequency one.

3.4 Magnetic

Accelerometers of this type are more common, and all have the same feature of depending on variations of magnetic flux linkages with a coil or a system of coils.[8] They exhibit many advantages including the comparatively small amount of mechanical energy that is usually required to operate them. This results in small mechanical loading effects. These electrical characteristics can be varied over a wide range permitting satisfactory operation with long connecting cables. However, although they give very good accuracy at low frequency ranges, they are severely limited in high frequency applications. The more common of the magnetic acceleration transducers are the

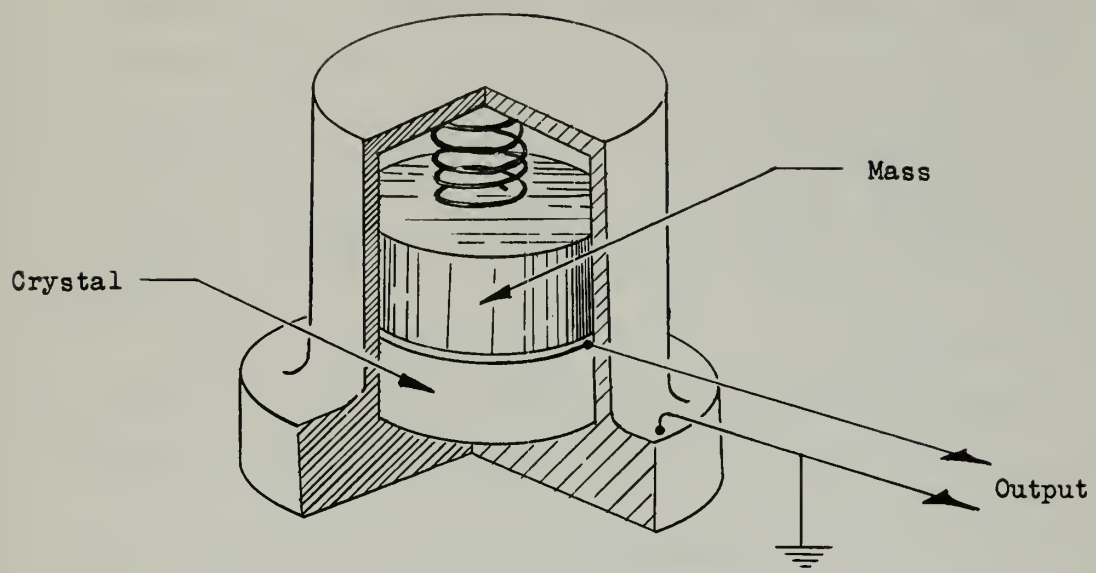


Fig. 3 - 1 Pictorial Diagram of a Piezoelectric Accelerometer

variable reluctance and the differential transformer types used in a suppressed carrier electronic system.

The variable reluctance accelerometer is as the name implies, based on any principle for changing the reluctances associated with magnetic circuits so that motion of the moving element causes flux lines to cut the turns of a coil system as they shift from one magnetic path to another.[8] They are typically constructed to contain two active inductive bridge arms. Each arm is wound with a magnetic shield which is provided with an air gap. This air gap is controlled in relationship to the applied acceleration by a moving element. The moving element is a seismically suspended magnetic armature.[20]

The characteristics of the variable reluctance transducer enable it to accept high accelerations and also enable it to have a high natural frequency. This is accomplished with an instrument that is very small in volume. These factors are desireable for this system.

In the case of the differential transformer accelerometer, the magnetic flux field varies by causing the permeability of the materials through which the field is conducted to change. These variations are induced differentially into several secondary coils which are mounted in a manner so as to oppose one another. The typical differential transformer accelerometer consists of a single primary and two secondary coils, connected as stated above in series opposition. Within the air gap of this air core transformer is a small seismically supported magnetic mass. The position of the mass is determined by the accelerations to which it has been subjected, which in turn controls the unbalance of the two output coils.[21] The characteristics of the differential transformer are in general less favorable although it does have a greater output sensitivity. Its acceptable maximum acceleration and natural frequency are

usually lower than a variable reluctance accelerometer of the same size.

It seems as if either accelerometer with the appropriate circuity could be used for this system. Both are small and lightweight, have acceptable output sensitivities and require excitation voltages and frequencies that are compatable with missile power systems. However, due to the higher natural frequency, and therefore the extended frequency range of the variable reluctance accelerometer it is believed that it is better of the two low frequency magnetic accelerometers for this system.

Various manufacturers such as, Gulton Industries[20][18] and North American Instruments, Incorporated[23][24] produce this type of accelerometer.

3.5 Typical High Frequency Accelerometer

The high frequency acceleration transducer that is to be normalized for use in this system is of the synthetic crystal piezoelectric type. Its acceptable range of characteristics, which is a composite of commercially available accelerometers, are as follows:[15][17]

Natural frequency:	> 4000 cps
Frequency range:	10 - 2000 cps
Sensitivity:	5 - 15 mv/g
Linearity:	$<$ ± 5 percent
Acceleration range:	$\geq$ 60 g's
Crosstalk:	$<$ 5 percent
Temperature range:	- 65° F to 200° F
Weight:	$<$ 3 oz.

Its equivalent circuit is:

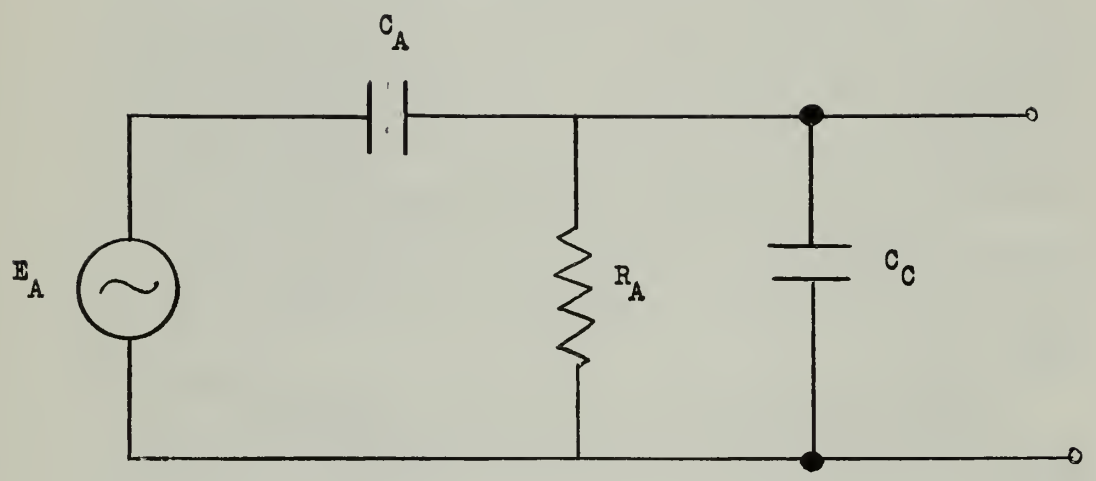


Fig. 3 - 2 Equivalent Circuit of a Piezoelectric Accelerometer

Typical values for the constants in Fig. 3-2 are:[26]

R_A	100 - 1000 megohms
C_A	300 $\mu\mu f$
C_C	100 $\mu\mu f/ft.$

In Fig. 3-2, E_A is a voltage generated proportional to the acceleration to which the accelerometer is subjected. C_A is the capacity, R_A is the effective load plus leakage resistance, and C_C is the effective cable capacitance. The low frequency cut off of the accelerometer is determined by C_A , C_C , and R_A ,

while the high frequency cut off is determined by its natural frequency. The natural frequency required of the high frequency accelerometer for this system is very low compared to those available commercially.

The typical performance curves are the results of these characteristics. (Fig. 3-3, 4, 5)

3.6 Typical Low Frequency Accelerometer

The variable reluctance acceleration transducer was selected for the low frequency accelerometer in this system. Its acceptable range of characteristics which is a composite of, and therefore compatable with, commercially available accelerometers, are as follows[20][21][23]:

Natural frequency:	30 - 300 cps
Frequency range:	0 - 100 cps
Excitation voltage:	1 - 25 volts
Excitation frequency:	400 - 5000 cps
Sensitivity:	0.5 - 20 $mv/g/V_{input}$
Linearity:	$< \pm 5$ percent
Acceleration range:	≥ 60 g's
Crosstalk:	< 5 percent
Weight:	< 3 oz.

The frequency range above represents the maximum that can be obtained with the maximum natural frequency listed. However a more limited frequency range may be used.

From static sensitivity tests of a typical low frequency accelerometer it was found that the output sensitivity in $mv/g/V_{input}$ varied linearly with carrier frequency to a

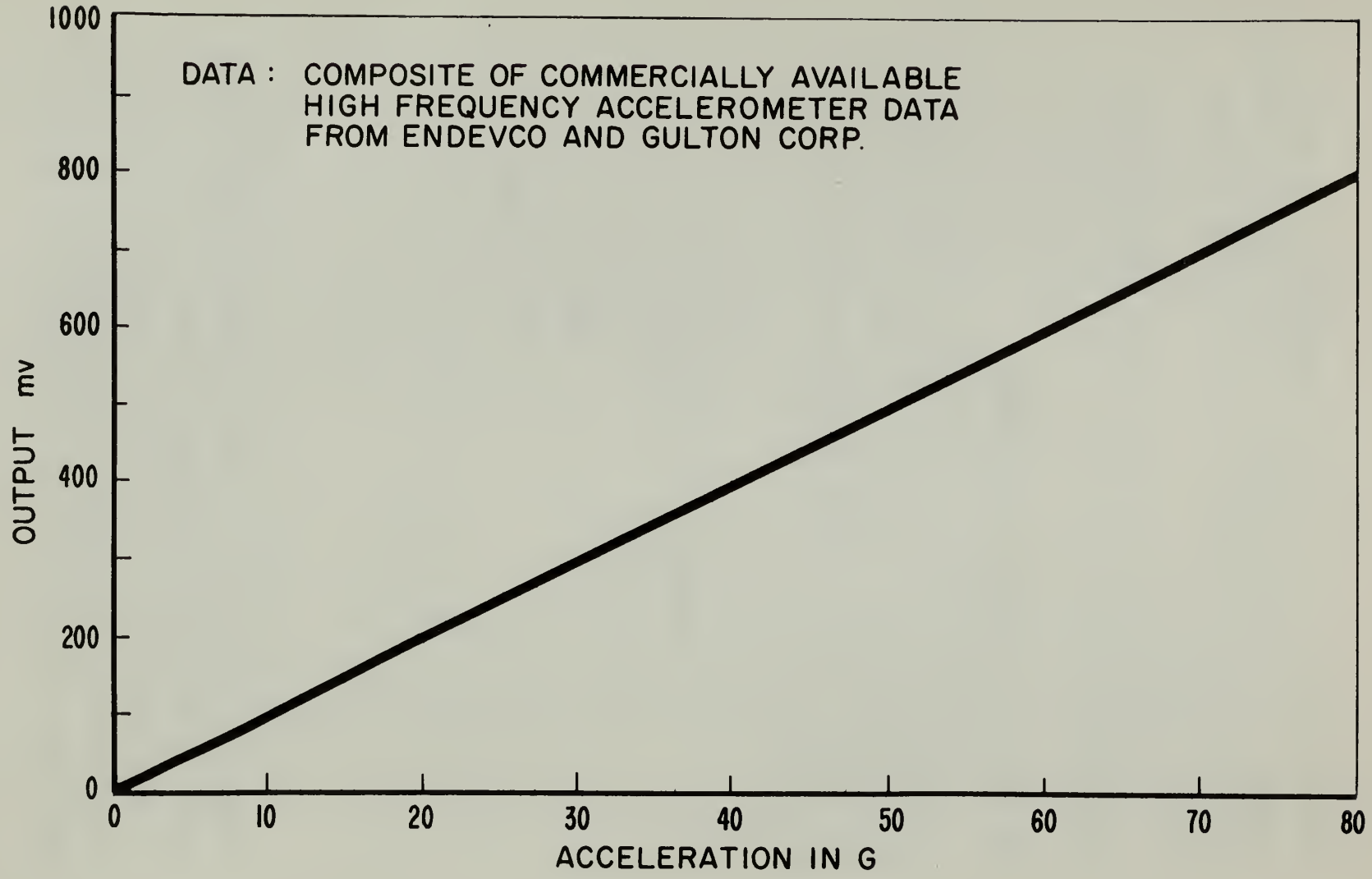


Fig. 3-3 Output vs. Acceleration for the Typical High Frequency Accelerometer

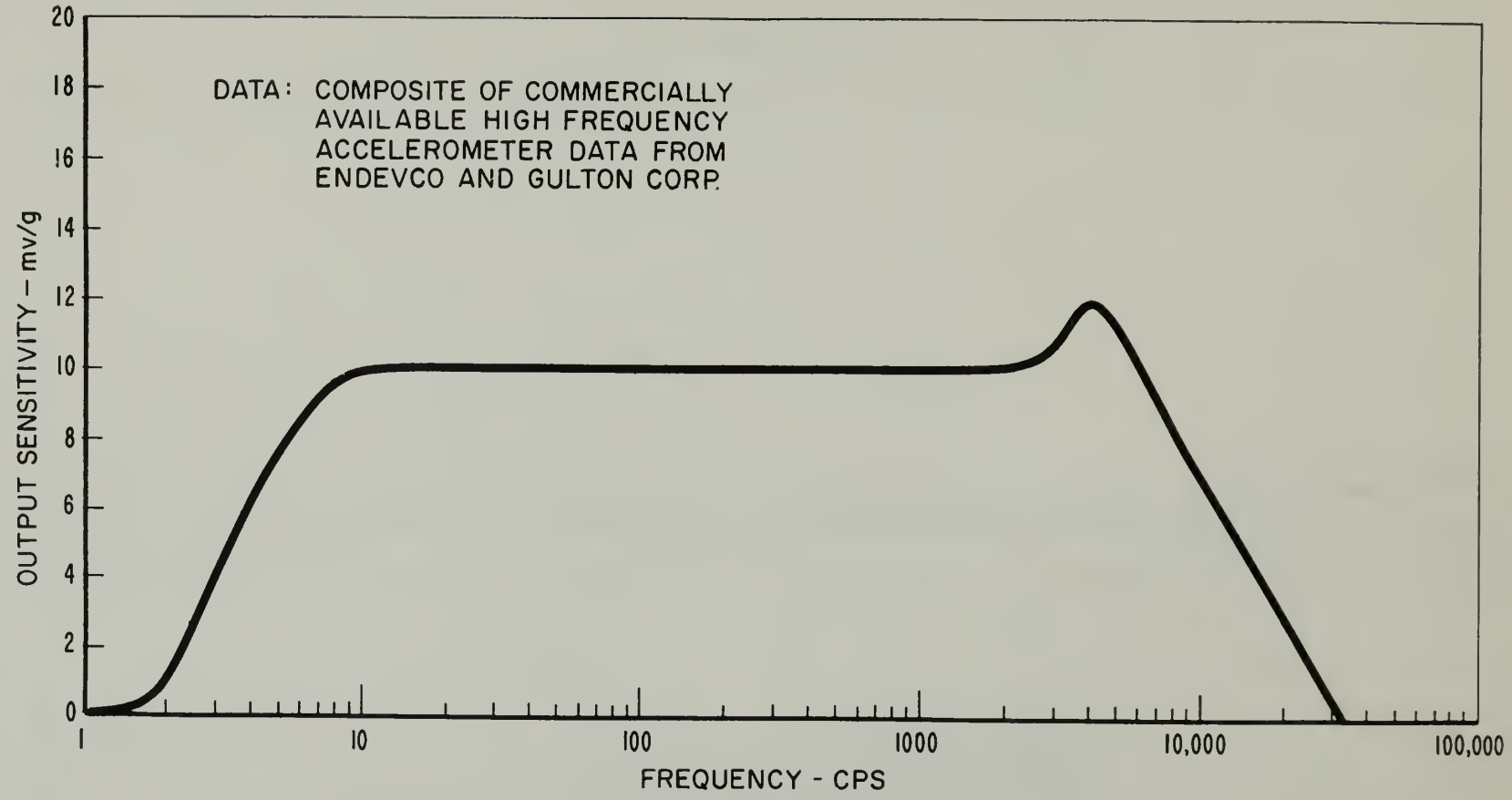


Fig. 3-4 Frequency Response for the Typical High Frequency Accelerometer

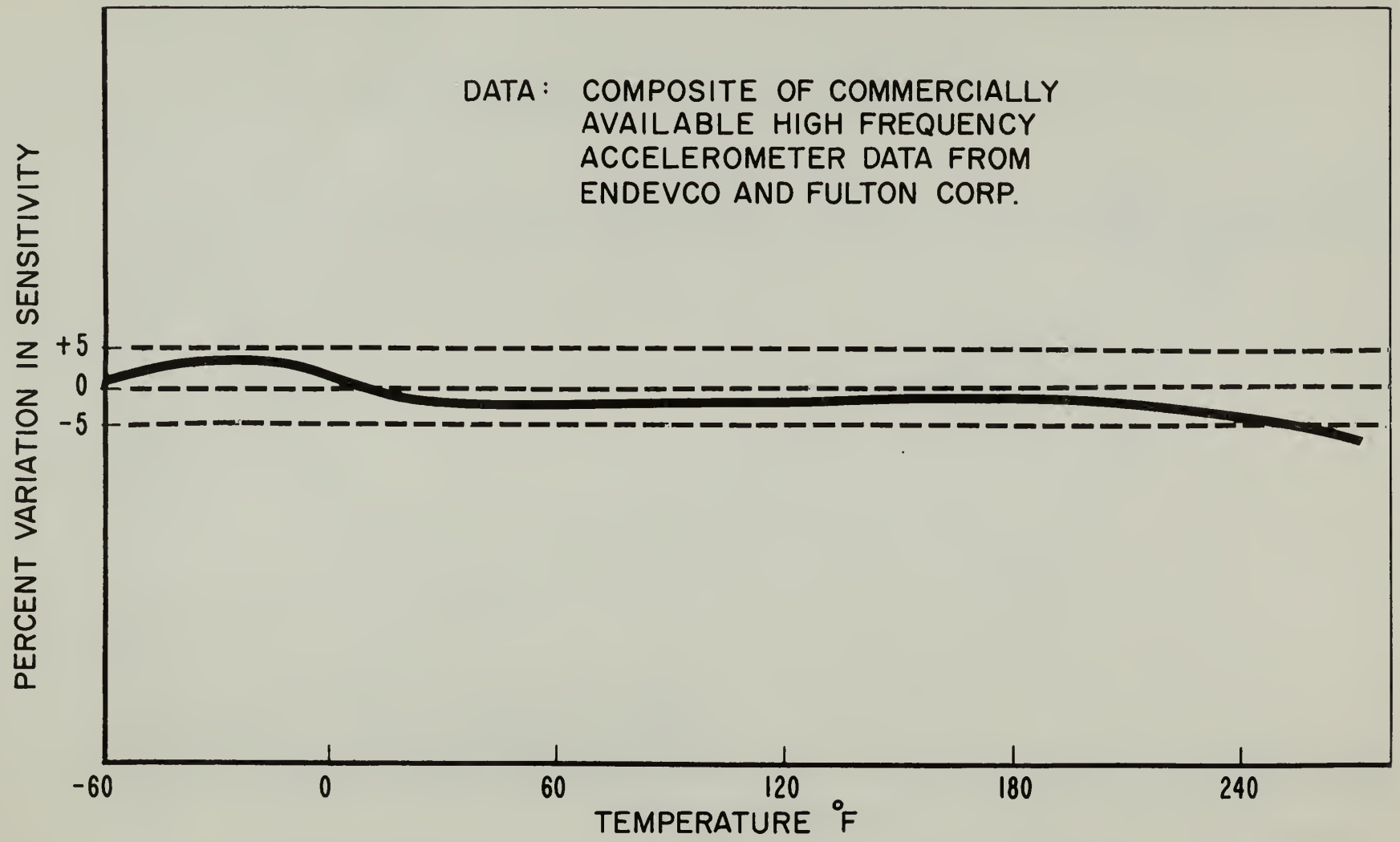


Fig. 3-5 Variation in Sensitivity vs. Temperature for the Typical High Frequency Accelerometer

maximum of 4000 cps. (Fig. 3-6). The data for Fig. 3-6 can be found in Appendix A.

These tests were conducted on a type A-10 accelerometer manufactured by Dynamic Measurements Co. and obtained from Test Equipment Sales Co..[29] An input acceleration of 1 g, and a constant input voltage of 6 volts was used.

Selecting a typical carrier frequency of 3000 cps; the resulting output was 17 mv/g. This value, used in connection with the typical characteristics, was the basis for the performance curves on the low frequency accelerometer. (Fig. 3-7, 8)

The typical carrier frequency value of 3000 cps was chosen because it, with its possible side bands, is safely above the required maximum acceleration frequency measured by this system. This fact is used as a basic parameter for another method of measuring and recording wide band accelerations, discussed in Chapter 6, Article 6.5.

With the range of characteristics now selected for both the high and the low frequency accelerometers, it appears that through the use of appropriate circuitry the frequency responses of both may be controlled and combined to obtain a flat response from 0 to 2000 cps. as in Fig. 3-9. Here, the output of each accelerometer is passed through the appropriate gain stage and high or low pass filter to obtain the indicated broadband flat response.

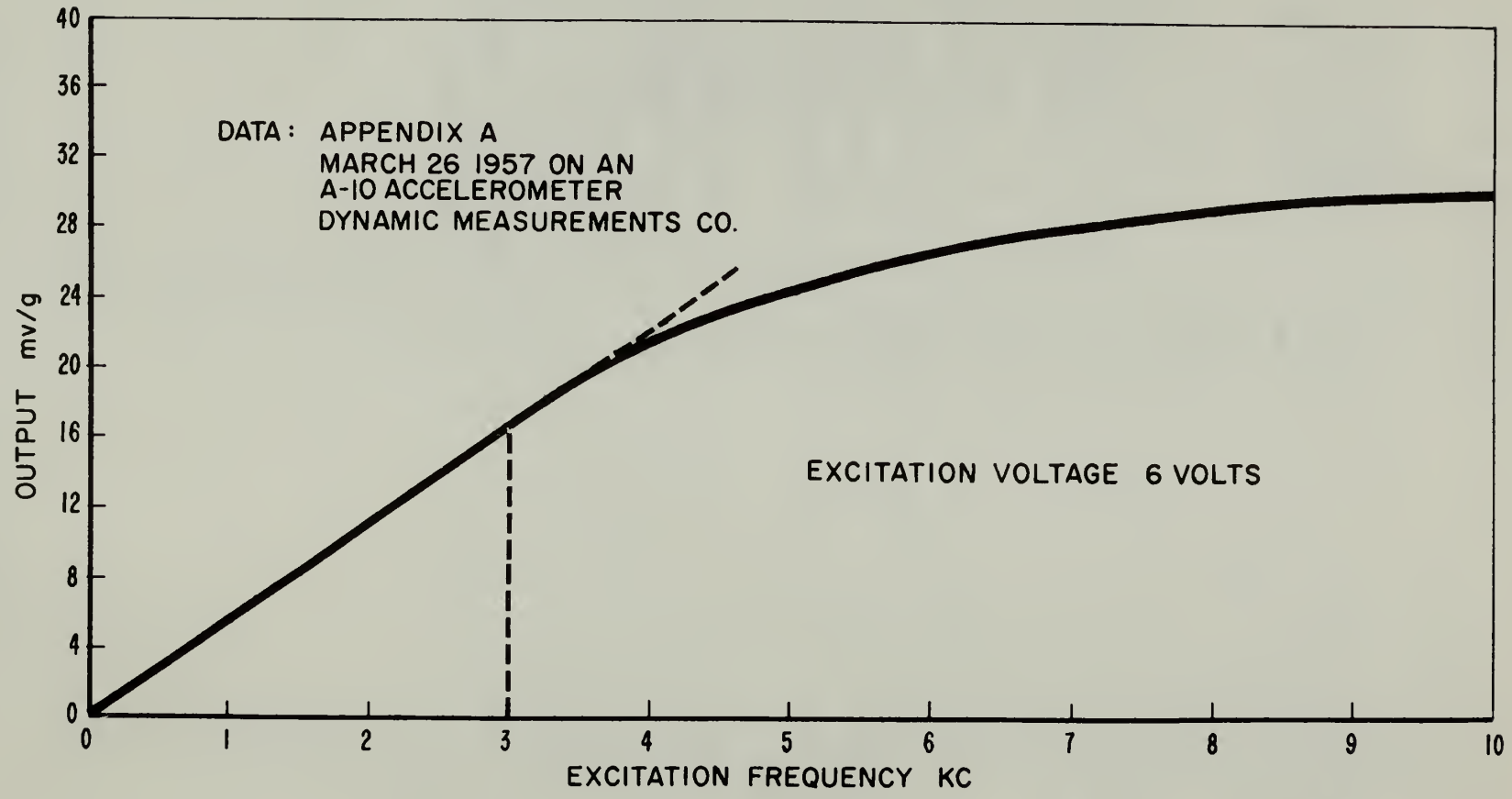


Fig. 3-6 Output vs. Excitation Frequency for the Typical Low Frequency Accelerometer

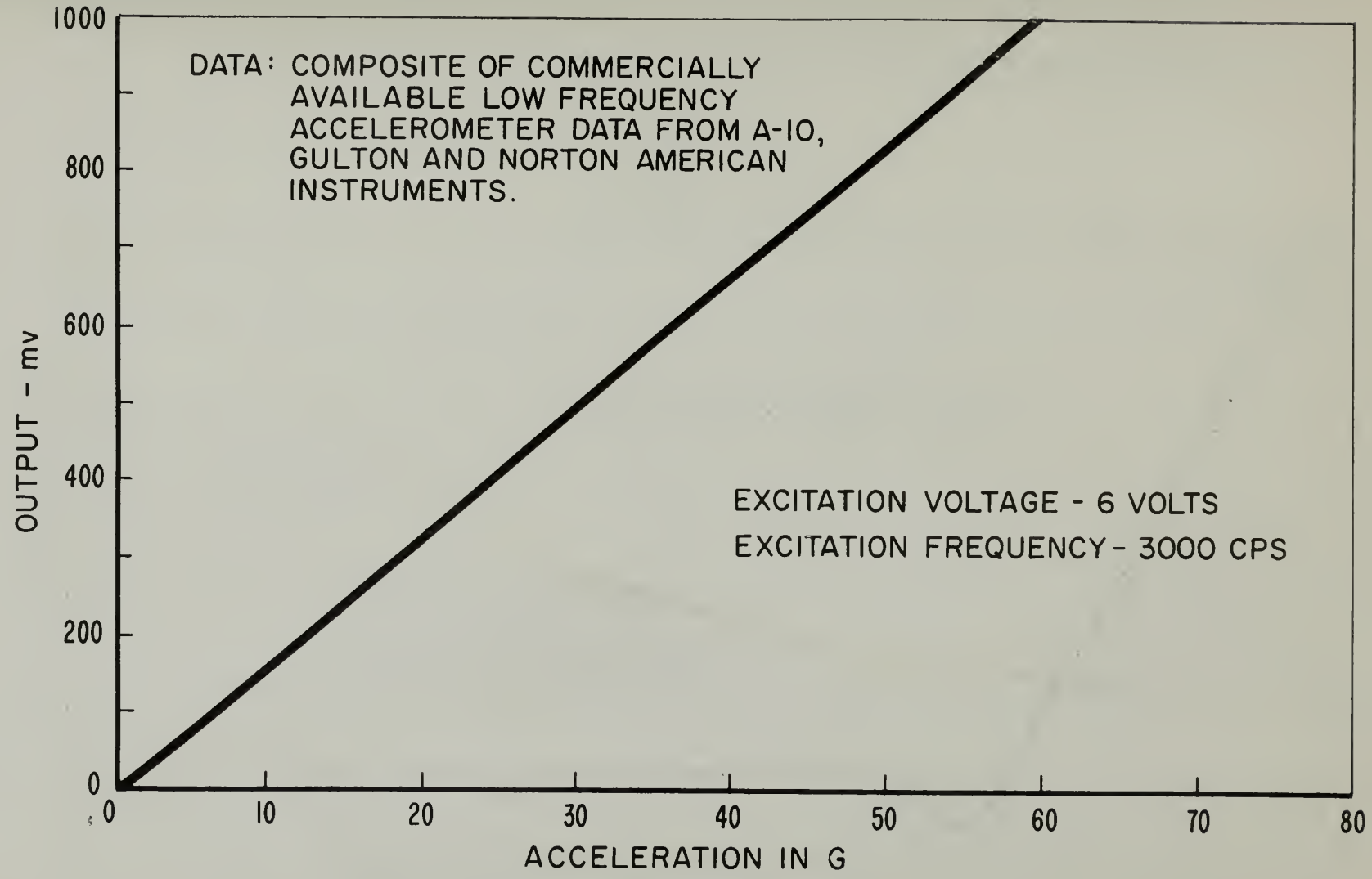


Fig. 3-7 Output vs. Acceleration for the Typical Low Frequency Accelerometer

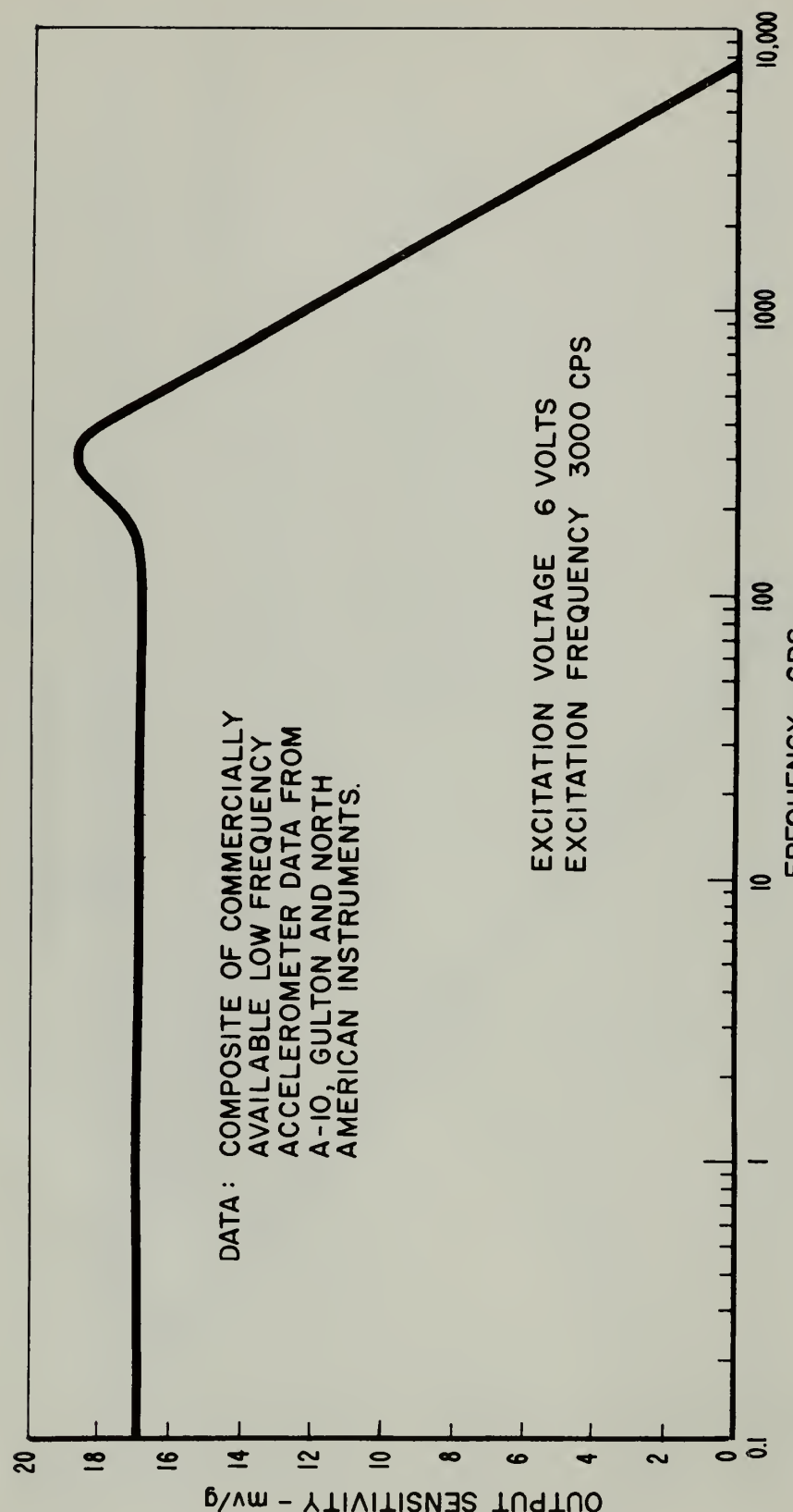


Fig. 3-8 Frequency Response for the Typical Low Frequency Accelerometer

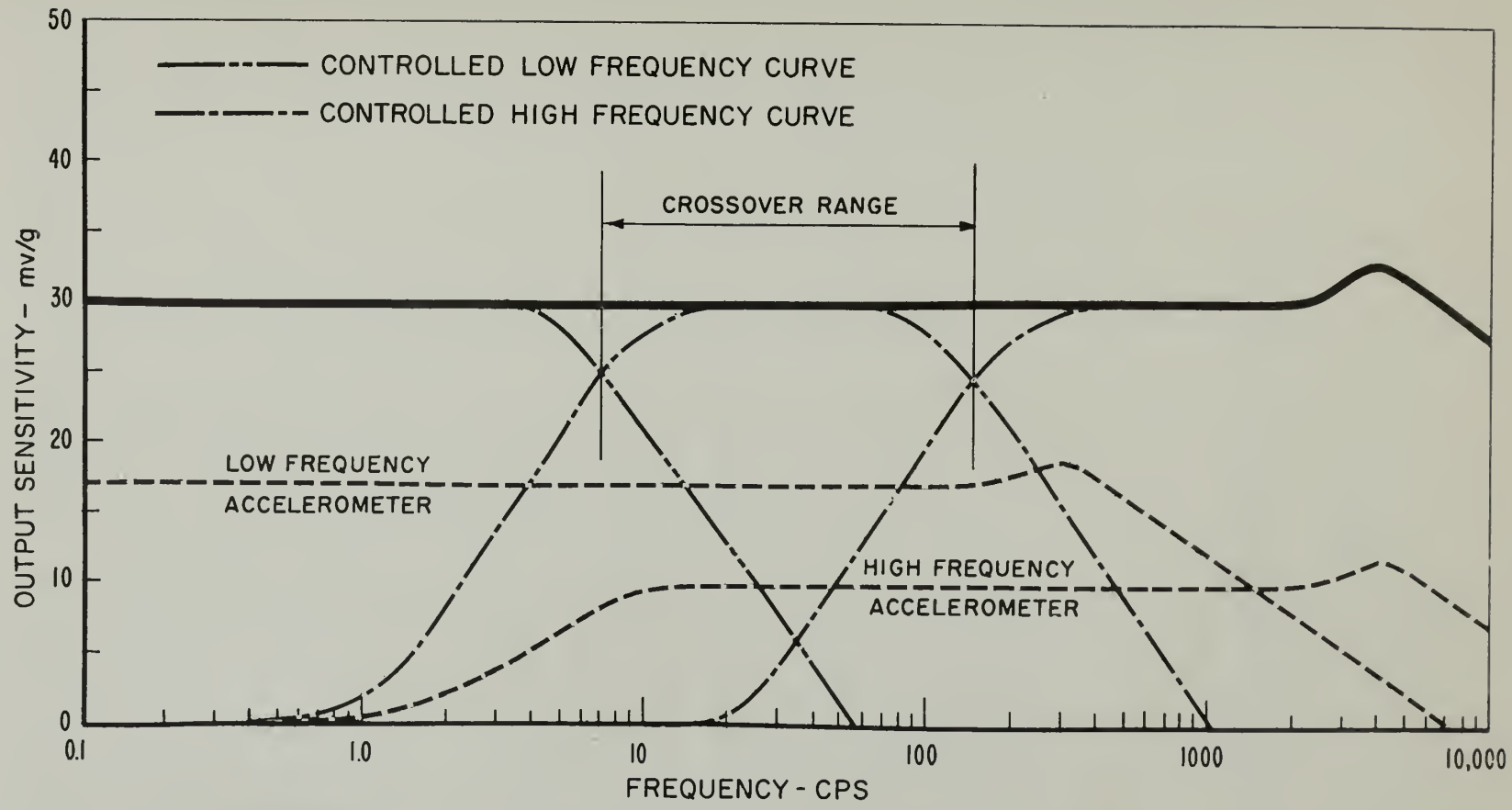


Fig. 3-9 Possible Normalized Frequency Response for Controlled Low and High Frequency Accelerometer

CHAPTER 4

WIDE BAND ACCELERATION MEASURING SYSTEM

4.1 General

The purpose of the measuring system is to combine the outputs of the selected high and low frequency accelerometers in such a manner as to give a flat frequency response over the desired range. This is done by controlling both the low frequency cutoff of the high frequency accelerometer and the high frequency cutoff of the low frequency accelerometer in such a manner that when the two outputs are added together a flat frequency response developes. This normalized signal is then to be controlled in such a way as to make it acceptable to telemetering and recording equipment, or in the absence of this link, acceptable to the analyzing system directly.

This chapter will specify the desired characteristics of the necessary components and how they should be tied together to give a normalized acceleration signal. (Fig. 4-1)

Noise will be introduced at various parts of the measuring system. One of the sources of this noise is from the connecting cables. The most common cause of noise in cables comes from the build-up of static charge on the outer shield as a result of friction. When the cable undergoes vibration, spurious voltages may be developed which could possibly be mistaken for acceleration signal. This is particularly true for the high frequency accelerometer. Therefore it is best to use short cables having a high flexibility, low capacitance, small size,

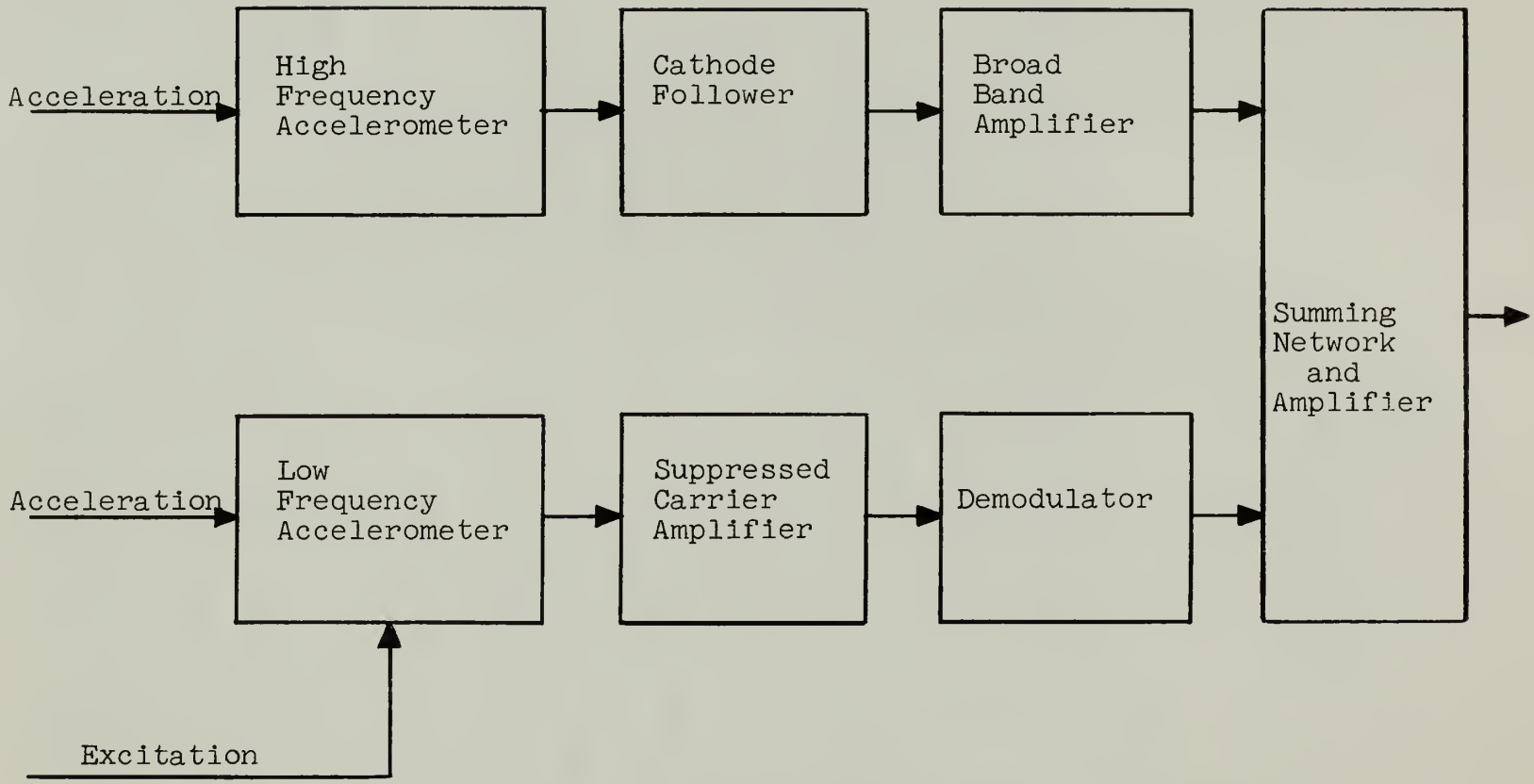


FIGURE 4-1 WIDE BAND ACCELERATION MEASURING SYSTEM

and good low noise characteristics.[30] Noise will also be introduced from the various electronic components. It is assumed in this study that noise has a flat power spectrum over the measured frequency range and therefore has equal weight throughout the measured acceleration power spectrum. It is also assumed that the level of this noise signal is small and this has little effect on the normalized output. However, all electronic components should have as low a noise figure as possible.

4.2 High Frequency Branch

The high frequency branch of this measuring system is composed of a high frequency accelerometer, a cathode follower and a wide band amplifier. (Fig. 4-2). The output from this branch is fed to a summing network, where it is combined with the output of the low frequency branch.

a) Accelerometer

Using a high frequency accelerometer whose characteristics stay within the bounds as set down in Chapter 3, but which has specific values obtained from a composite of this type of accelerometer, for these characteristics, is the starting point for this branch. These specific values are as follows:

Natural Frequency:	4000 cps
Frequency range:	10 - 2000 cps
Sensitivity:	10 mv/g
Acceleration range:	60 g's

The output of this accelerometer is then fed by a cable to a cathode follower. The selection of this cable, as stated before, will be a factor if accurate readings are desired.

b) Cathode Follower

The cathode follower is in the high frequency branch

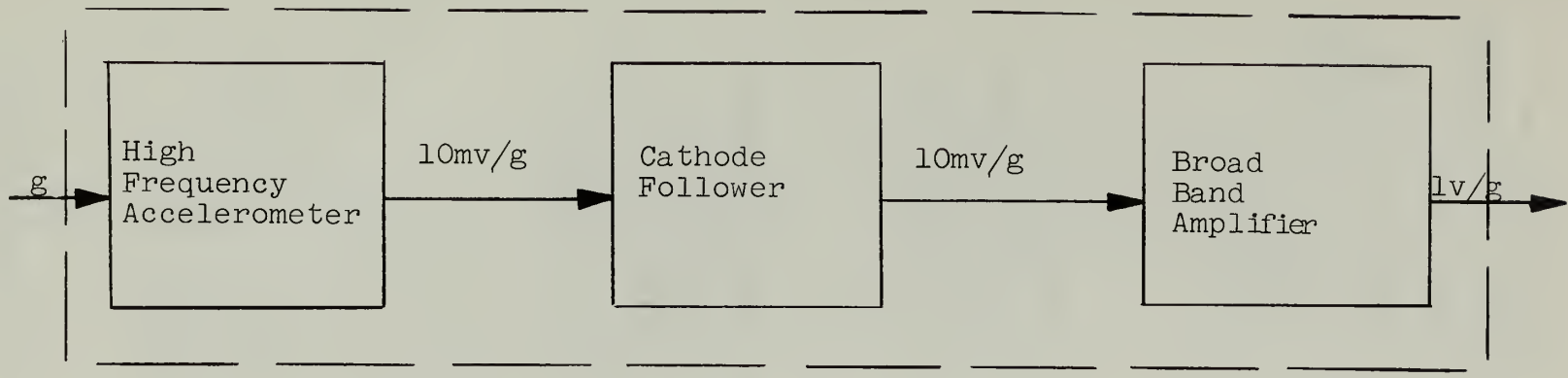


FIGURE 4-2 HIGH FREQUENCY BRANCH OF MEASURING SYSTEM

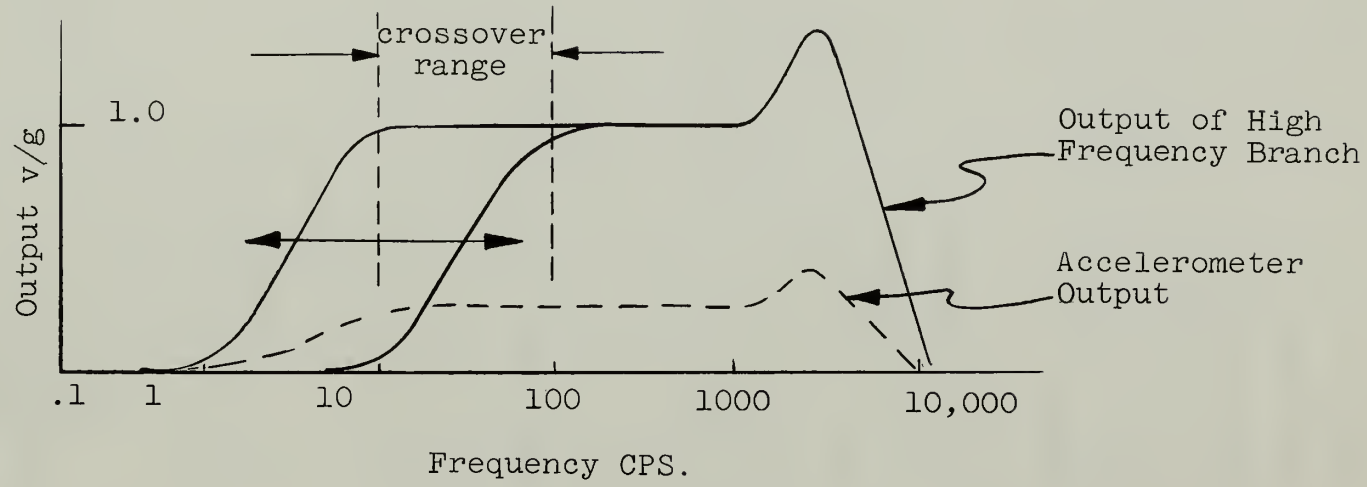
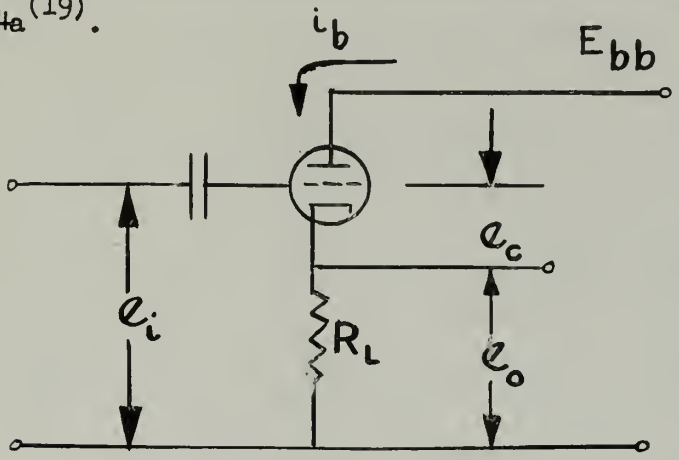


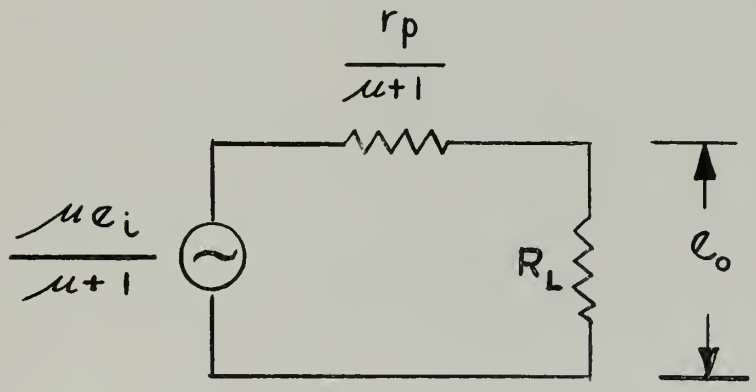
FIGURE 4-3 DESIRED CONTROL OF FREQUENCY RESPONSE OF HIGH FREQUENCY BRANCH

to supply the necessary input impedance to maintain a minimum low frequency cutoff point for the piezoelectric accelerometer, if it is desired.

The basic circuit of the cathode follower appears in Fig. 4-4a[19].



a) Basic Circuit



b) Voltage Source Equivalent

Fig. 4 - 4 Basic Cathode Follower

In this figure E_{bb} is the plate supply voltage, e_i is the input voltage, e_o is the output voltage, e_c is the grid to cathode voltage, i_b is the plate current and R_L is the load resistance. With an increase in e_i, i_b tends to increase

causing an increase in e_o. Therefore, when operating within the linear range of the tube characteristics a change in the input voltage gives a corresponding change in the output voltage.(31)

The amplification of the cathode follower is always less than unity because the change in e_o requires a change of e_c to produce it and the change of e_i is the sum of the changes of e_c and e_o. This can be seen from a voltage source equivalent diagram of the cathode follower, Fig. 4-4b. In this figure, μ is the amplification factor of the tube and r_p is the incremental resistance of the tube. The voltage amplification e_o / e_i, approaches $\mu / \mu + 1$ as the load impedance becomes large. The cathode follower is therefore not useful as a voltage amplifier. But it finds extensive use as an impedance transformer between a source having a high internal impedance and a load having a low impedance.

The typical cathode follower circuit as used with the accelerometer in this high frequency branch is shown in Fig. 4-5. In this circuit, the grid return resistor, R_g, is inserted into the cathode circuit so that the current developed in the cathode circuit applies feedback into the input circuit. The net result is a multiplication of the input impedance of the amplifier by the value of R_g.(34)

The characteristics of the cathode follower should be:

Frequency range:	1 - 5000 cps
Gain:	$\geq$ 0.94
Input impedance:	50 - 100 megohms
Output impedance:	100 - 300 ohms
Noise level (rms):	100 microvolts

All of the above characteristics are available in many commercial cathode followers, one of which is made by Gulton

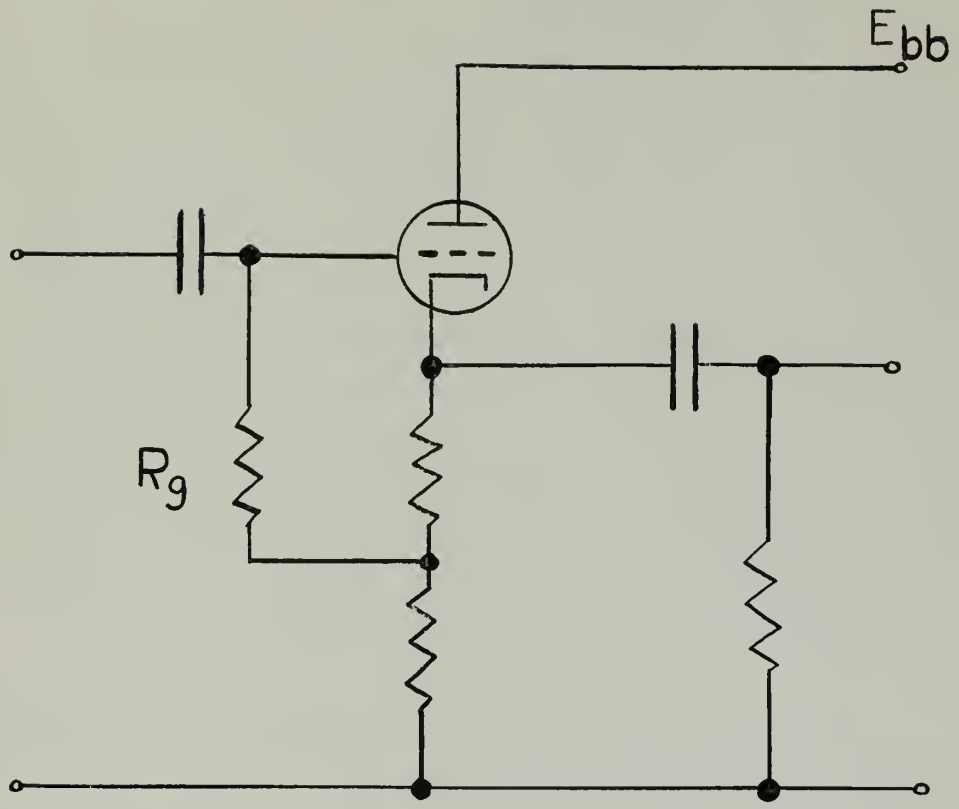


Fig. 4 - 5 Typical Cathode Follower Circuit Used with the Piezoelectric Accelerometer

Manufacturing Corporation.[18] However, the frequency range of commercial cathode followers is usually much greater than needed for this system.

Assume now an input signal to the cathode follower of 10 mv/g over a frequency range of 10 - 2000 cps. The output of the cathode follower, assuming a gain of 1 , would match the input.

This signal is then fed to a wide band amplifier.

c) Wide Band Amplifier

The wide band amplifier must accept the output of the cathode follower and control it in such a manner as to present an acceptable signal to the summing network. Considering the summing network as a summing device only, its gain equal to 1, the wide band amplifier must control both the low frequency cutoff and the gain of the cathode follower signal. The low frequency cutoff must be controlled in such a way as to vary it over the crossover range. In this system the crossover range is approximately 10 - 100 cps. The gain of the wide band amplifier must be sufficient to increase the signal to that level required as an output from the measuring system. This output signal from the complete measuring system, referred to as the normalized acceleration signal, should have a value of 1 v/g. (Fig. 4-3)

A normalized signal of 1 v/g was selected because of its compatibility with the remaining systems.

The wide band amplifier that seems to comply with all of the above requirements is the resistive-capacitance coupled amplifier. It is very stable, inexpensive due to its simplicity, and has the required frequency characteristics.[19]

Its lower half-power frequency is controlled by the coupling capacitance, while its upper half-power frequency is controlled by its shunt capacitance. (Fig. 4-6a)

The gain is controlled at the low frequency end mainly by the grid resistance and in the middle and high frequency region by the load resistance. (Fig. 4-6b, 6c)

Therefore through adjusting various capacitances and resistances the frequency response of this amplifier can be made to cut off properly.[19]

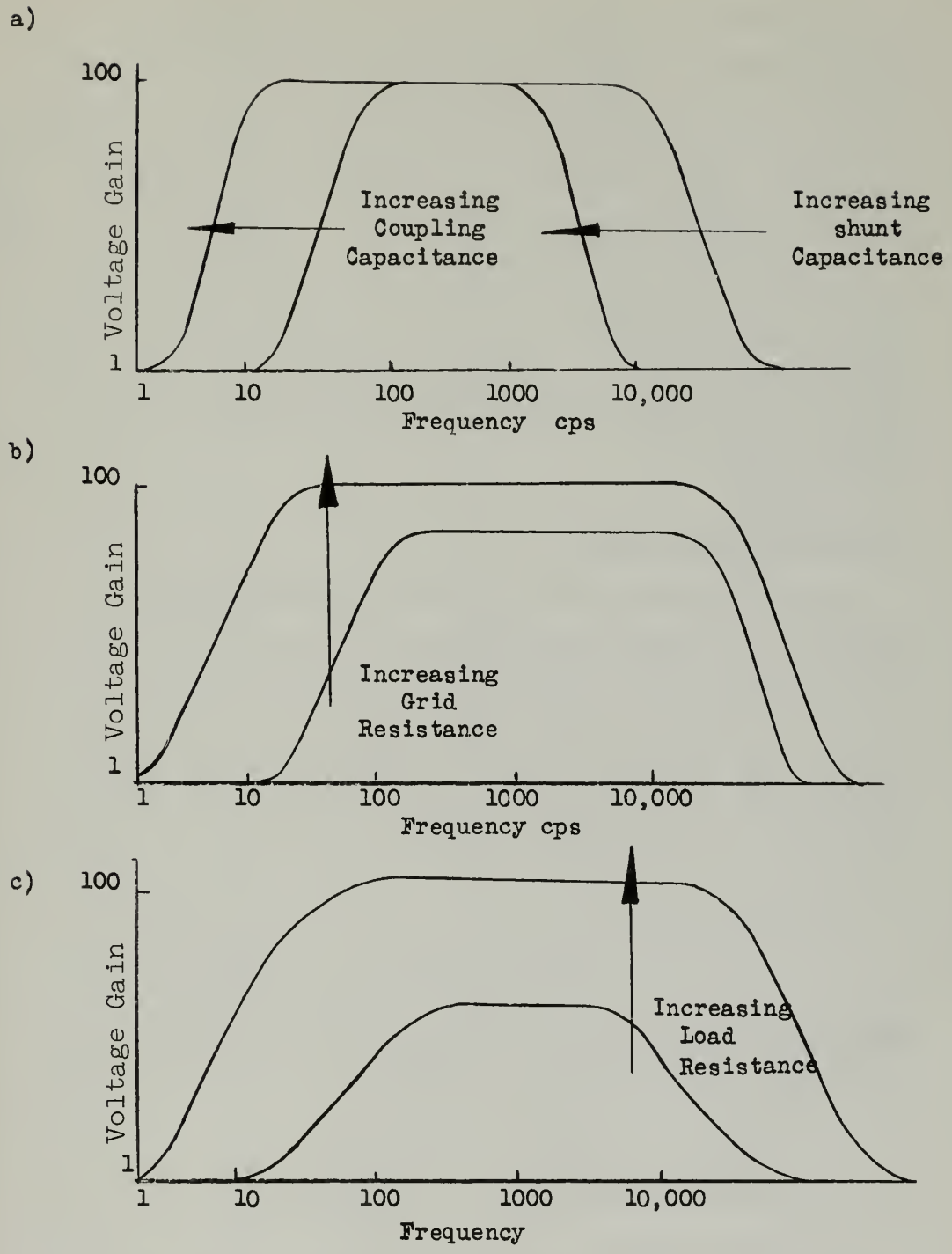


Fig. 4 - 6 Control of Frequency Response of Resistive-Capacitance Coupled Amplifier

Considering the input signal to this amplifier and the desired output of 1 v/g, the amplifier must be able to shift the low frequency cutoff point through a range of 10 to 100 cps and must have a voltage gain of at least 100, over a frequency range of 5 - 5000 cps. (Fig. 4-3) These characteristics are found in commercial amplifiers.

However, if it is desirable that the wide band amplifier control only the gain of the acceleration signal, then the low frequency cutoff must be controlled by another component. This component could be a simple high pass filter employed in the summing network.

4.3 Low Frequency Branch

The low frequency branch of this measuring system is similar to the high frequency branch. It contains the low frequency accelerometer, a suppressed carrier amplifier, a demodulator, and a low pass filter. (Fig. 4-7) The low pass filter can be included in the summing network and its output combined with the output of the high frequency branch to give the normalized acceleration signal.

a) Accelerometer

The low frequency accelerometer has characteristics that are similar to the typical one discussed in Chapter 3. Its specific characteristics, obtained from a composite of this type accelerometer are as follows:[20]

Natural frequency:	300 cps
Frequency range:	0 - 100 cps
Sensitivity:	20 mv/g
Acceleration range:	60 g's

Its excitation voltage and frequency are assumed to be sufficient to give a sensitivity of 20 mv/g.

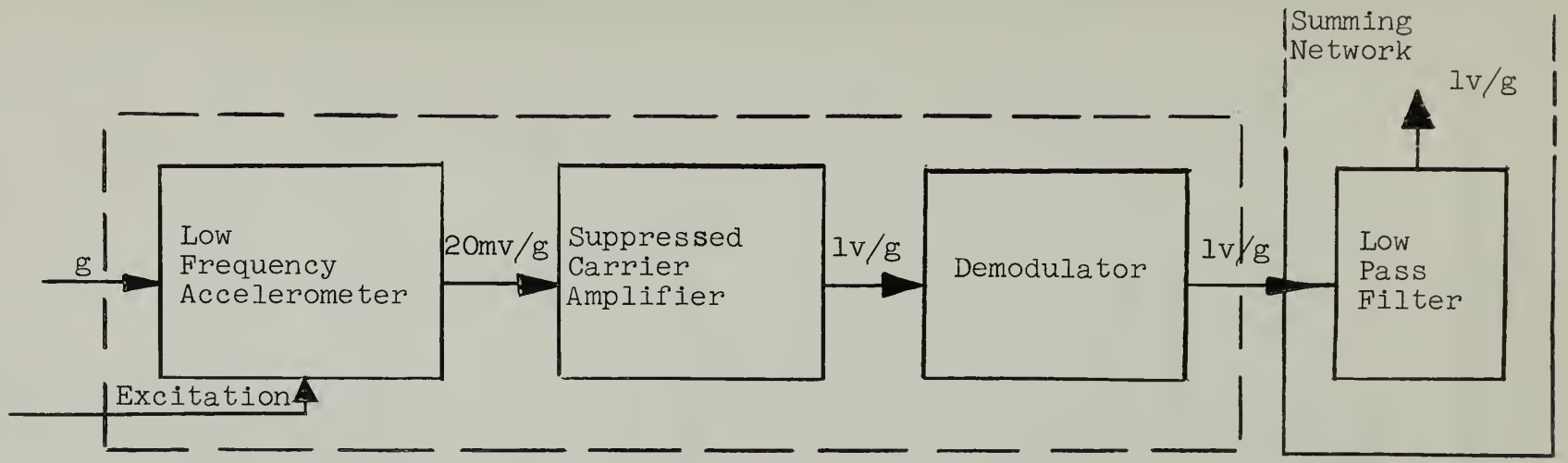


FIGURE 4-7 LOW FREQUENCY BRANCH OF MEASURING SYSTEM

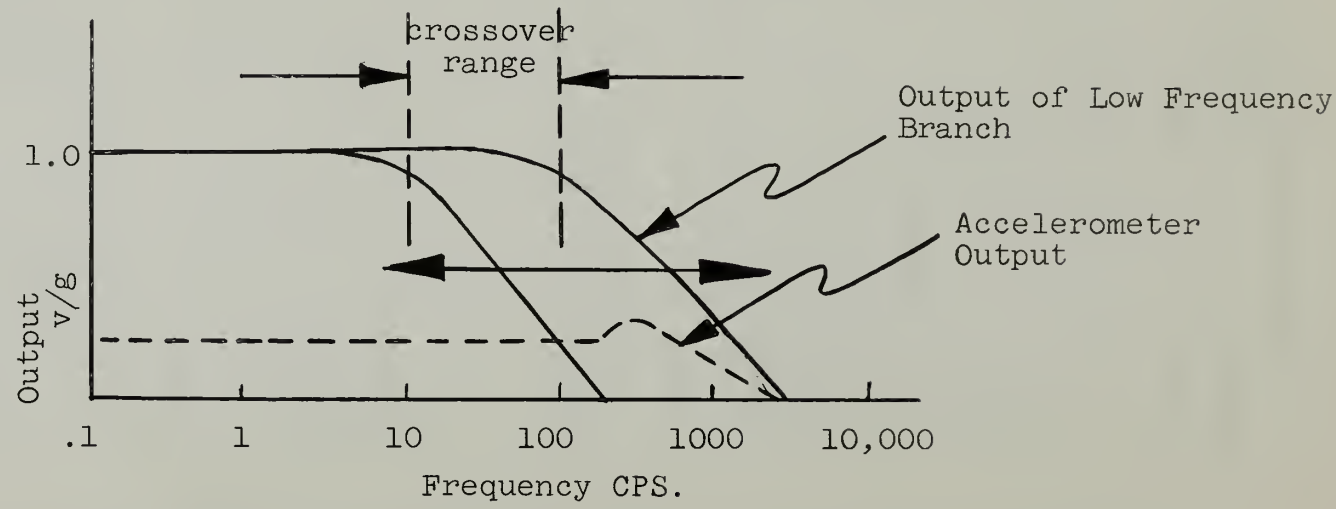


FIGURE 4-8 DESIRED CONTROL OF FREQUENCY RESPONSE OF LOW FREQUENCY BRANCH

The output of this low frequency accelerometer is a suppressed carrier signal. A suppressed carrier amplifier is therefore used to amplify the signal.

b) Suppressed Carrier Amplifier

In order to bring the low frequency accelerometer signal of 20 mv/g up to a level where it can be combined satisfactorily with the output of the high frequency branch a gain of 50 is desired for the suppressed carrier amplifier. The amplifier should also have low noise characteristics, and have a linear response over the range of input frequencies. These characteristics are found in many types of commercially available strain gage amplifiers. These amplifiers are basically R - C coupled.

This amplified signal having a magnitude of 1 v/g must then be fed to a demodulator.

c) Demodulator

The demodulator is present to transfer from a carrier reference to a zero frequency reference, the output of the low frequency branch accelerometer. This is done to obtain a pure acceleration signal.

This acceleration signal is then passed to a low pass filter.

d) Low Pass Filter

The low pass filter controls the high frequency cutoff of the low frequency accelerometer. (Fig. 4-8, 9) This type of filter is discussed in Chapter 5, Article 5.3.

The frequency range through which this cutoff point should be controlled for this system is 10 - 100 cps which is the crossover range. (Fig. 4-8)

The output of the low pass filter is then combined with the output of the high frequency branch.

4.4 Summing Network and Amplifier

The outputs of both the high frequency branch and the low frequency branch have the same signal magnitude of 1 v/g. They are also controlled by their various circuitry in such a manner as to match the low frequency cutoff of the high frequency accelerometer with the high frequency cutoff of the low frequency accelerometer. Combined in this fashion it is believed that a flat frequency response will be obtained over the desired range of 0 to 2000 cps. This is the normalized acceleration signal and the output of the proposed measuring system. (Fig. 4-10)

The amplifier is to be a chopper stabilized wideband d-c operational amplifier capable of driving the analyzing system or the transmitting link at its output.

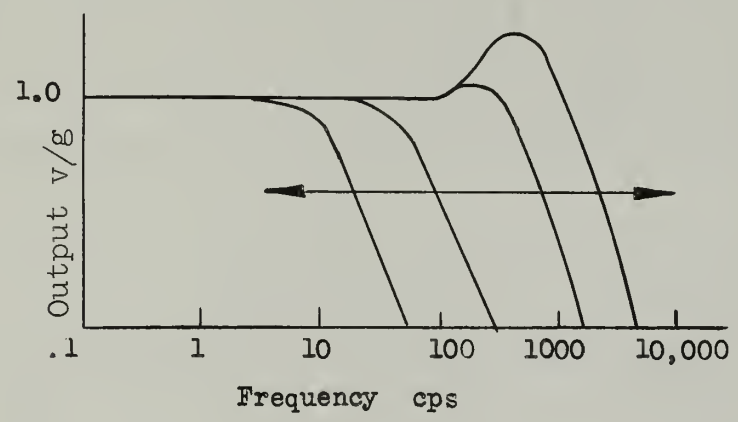


Fig. 4 - 9 Control Desired by Low Pass Filter

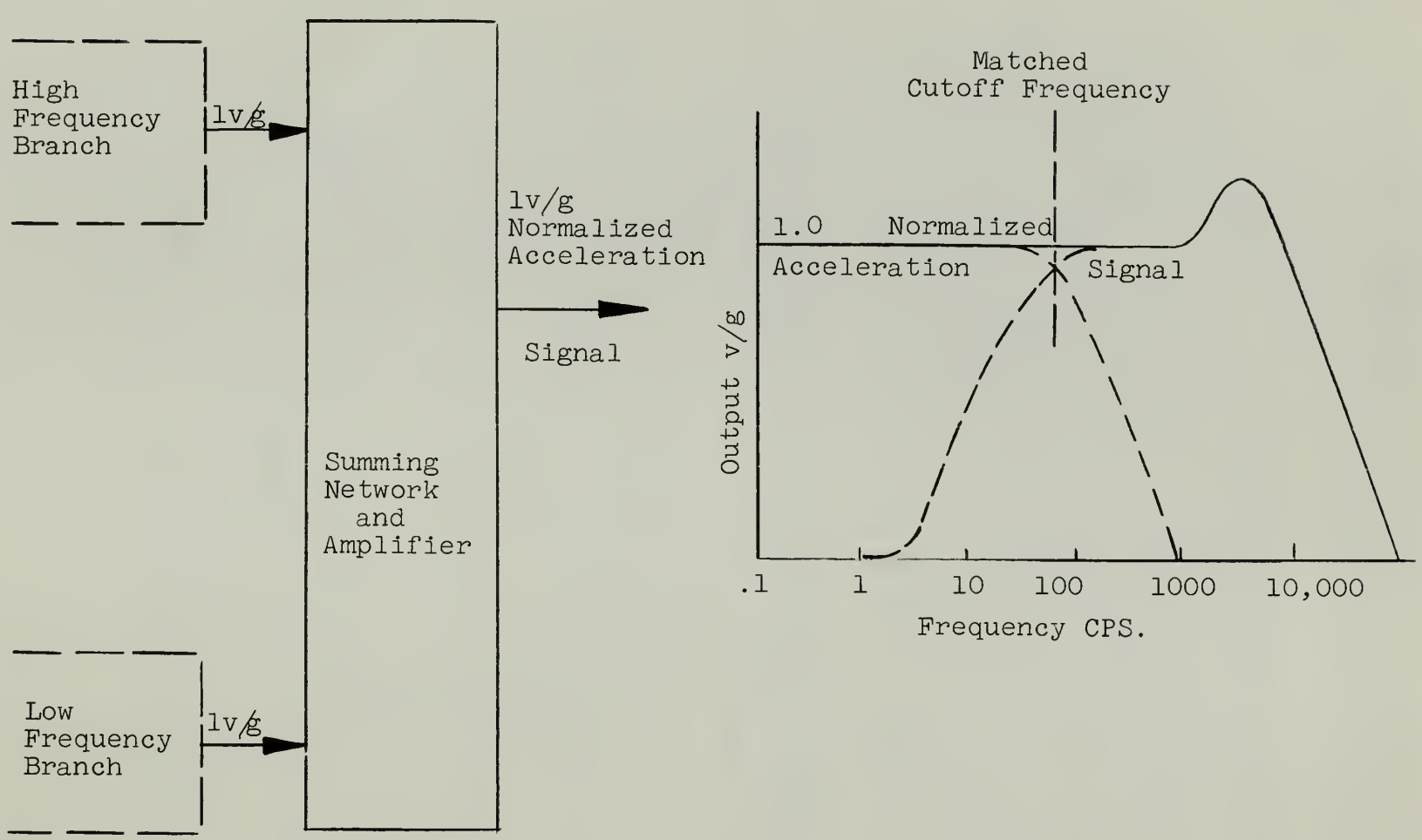


FIGURE 4-10 SUMMING NETWORK

CHAPTER 5

ANALYZING SYSTEM

5.1 General

The transmission and receiving circuits of the telemetering system used on the missile will not be discussed. They are assumed to be adequate to retain the required signal intelligence. However, the recording device will be discussed as a connecting link between the measuring system and the analyzing system. (Fig. 5-1)

5.2 Recording Device

The frequencies of interest contained in the normalized acceleration signal are from 0 to 2000 cps. This signal, if fed directly to a recording system, must be recorded in such a manner that the signal may be reproduced as often as desired. The reproduced signal must be as perfect a representation of the original as desired and useable.

The magnetic tape recorder, as presently available, will most closely fulfill this basic requirement. The recorder must have a flat response over the range of 0 to 2000 cps. It must have low noise and hum levels and very little signal distortion. The recorder must also run at constant speed in order that the frequencies will not be distorted in this manner.

There are magnetic tape recorders available which use a special frequency modulated carrier technique. They have been used successfully in low frequency vibration work.[13] Multi-channel types are available so that the acceleration signals

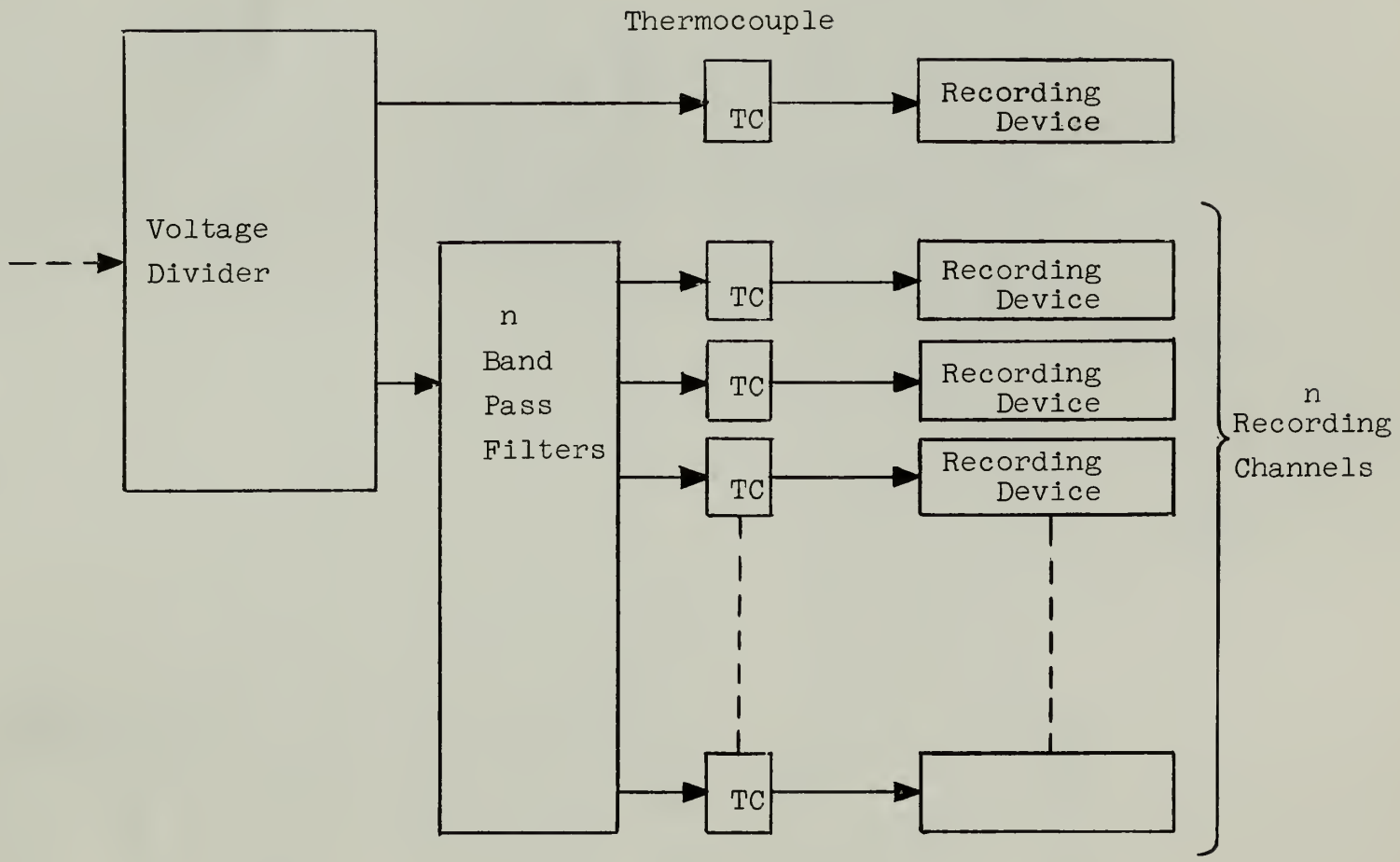


FIGURE 5-1 WIDE BAND ANALYZING SYSTEM

from the three axes of interest may be recorded simultaneously.

The Ampex Magnetic Recorder[14] - Model 306 has characteristics which meet the requirements previously stated. Its frequency response is similar to that presented in Fig. 5-2. Its tape speed is 30 inches per second and maximum recording time is 16 minutes.

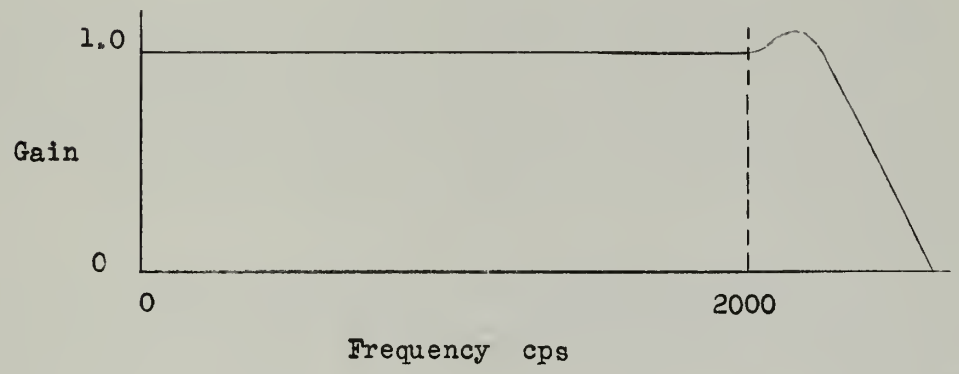


Fig. 5 - 2 Recorder Gain vs. Frequency

The Ampex FR-100, also available, has a tape speed of 15 inches per second and a running time of 32 minutes. By using a different type of tape which is thinner more can be stored on a reel. The recording time in this manner can be extended to 48 minutes, which exceeds the required specification of 30 minutes. The phase change is considered unimportant because phase information is lost in the thermocouple measurement.

5.3 Bandpass Filters

An ideal bandpass filter can only be approximated in actual practice and at best the approximation is imperfect. However, by suitable choice of crossover points, the approximations are sufficiently valid for the purposes of the proposed system.

Filters are intended to operate at or near a center frequency (ω_i) and should pass one hundred percent of the power present at that frequency. The filter should also pass one hundred percent of the power present at frequencies in the filter bandwidth. This is done only in theory by ideal filters.

There is a power loss (attenuation) in the side bands, i.e., $\omega_i \pm \Delta\omega$, of practical filters. Usually, this loss increases the further ω is from ω_i. The frequencies at which the attenuation if fifty percent are called the "half power frequencies."(19) The range between these frequencies is called the filter "bandwidth."

It has been found that in using filters to cover an extensive range best results are obtained by matching the half power frequencies. This method has been used in designing the filters used in this discussion.

A simple R-L-C circuit operating as a low pass filter has the following characteristics. (Fig. 5-3).

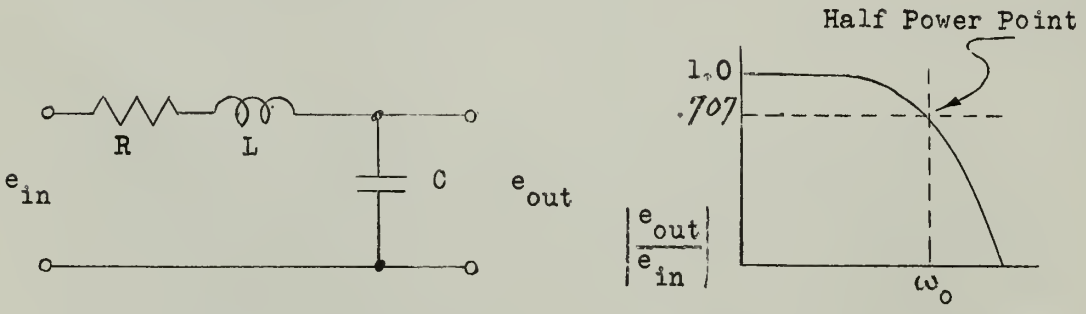


Fig. 5 - 3 R-L-C Circuit and Its Frequency Response

$$\frac{e_{out}}{e_{in}} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 - \omega^2 LC + j\omega RC} \sim \frac{1}{1 - \frac{\omega^2}{\omega_o^2} + \frac{2j\zeta\omega}{\omega_o}} \qquad [5\text{-}1]$$

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad [5\text{-}2]$$

$$\frac{2\zeta}{\omega_o} = RC \qquad [5\text{-}3]$$

The simple R-L-C filter can be modified somewhat to change the center frequency yet retain the fixed bandwidth of ω_o.[6] In this case Q, which is a figure of merit in filter design, varies directly as the center frequency.

$$Q = \frac{\omega_i}{\omega_o} \quad \frac{\text{center frequency}}{\text{bandwidth}} \qquad [5\text{-}4]$$

It is possible to keep Q a constant but then it would be necessary to change bandwidth to keep the ratio constant.[22]

The AC equivalent of the low pass R-L-C circuit with the same total bandwidth is seen in Fig. 5-4.

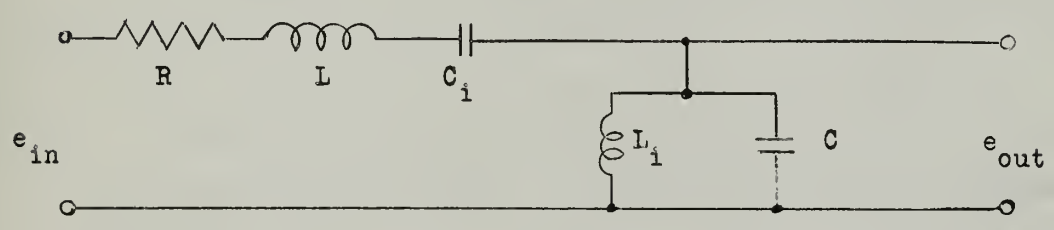
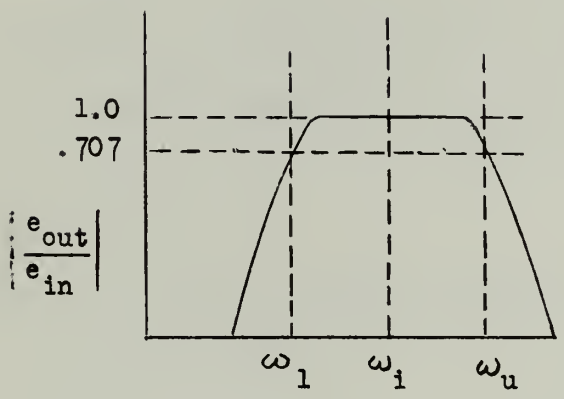


Fig. 5 - 4 AC Equivalent Circuit

$$C_i = \frac{1}{\omega_i^2 L} \qquad [5\text{-}5]$$

$$L_i = \frac{1}{\omega_i^2 C} \qquad [5\text{-}6]$$

The frequency response of this circuit is approximately that in Fig. 5-5



ω_l = lower half power point
ω_u = upper half power point
ω_i = center frequency

Fig. 5 - 5 AC Filter Frequency Response

and $(\omega_u - \omega_l)$ equals ω_o, the constant bandwidth.

Appendix B contains the design specifications and computations. A plot is presented in Fig. B-1 which shows how the frequency response varies with ζ. On the frequency response plot the point at which $\left| e_{OUT}/e_{IN} \right|$ = .707 is the half power point. The ζ which came closest to positioning the half power points at the required frequencies was .72 ; ζ depends

on the fixed R of the filter. Therefore, adjustments can be made on R , in testing and calibration, in order to give the most satisfactory frequency response in conjunction with adjoining filters.

It is concluded that bandpass filters can be made and properly spaced so that their summed results satisfactorily approximate results expected from the ideal filters. (Fig. 5-6)

5.4 Thermocouple

The vacuum thermocouple is a square law current measuring device. It is a thermoelectric junction and a heater wire mounted in a small, highly evacuated glass bulb. The junction can either be connected directly to the heater or separated from it by a thin coating of insulating material. This insulation aids in protecting the thermocouple which can easily be overloaded and thus burned out and, further, allows the convenience of isolated input and output terminals. Evacuating the glass bulb prevents air circulation due to convection currents and the accompanying loss of heat.[25] Thus, accuracy and sensitivity are increased.

Measurement is accomplished by passing the AC current through the heater which increases the temperature of the thermoelectric junction. A DC voltage is generated in the junction which is proportional to the temperature rise and to the square of the impressed current. In the proposed system this generated voltage is measured by a recording galvanometer.

a. Limitations

The voltage generated is proportional to the square of the current in the heater but only for small values of current. As current is increased beyond small values radiation occurs and the voltage out no longer follows the square law.[25] This can be seen in Fig. 5-7.

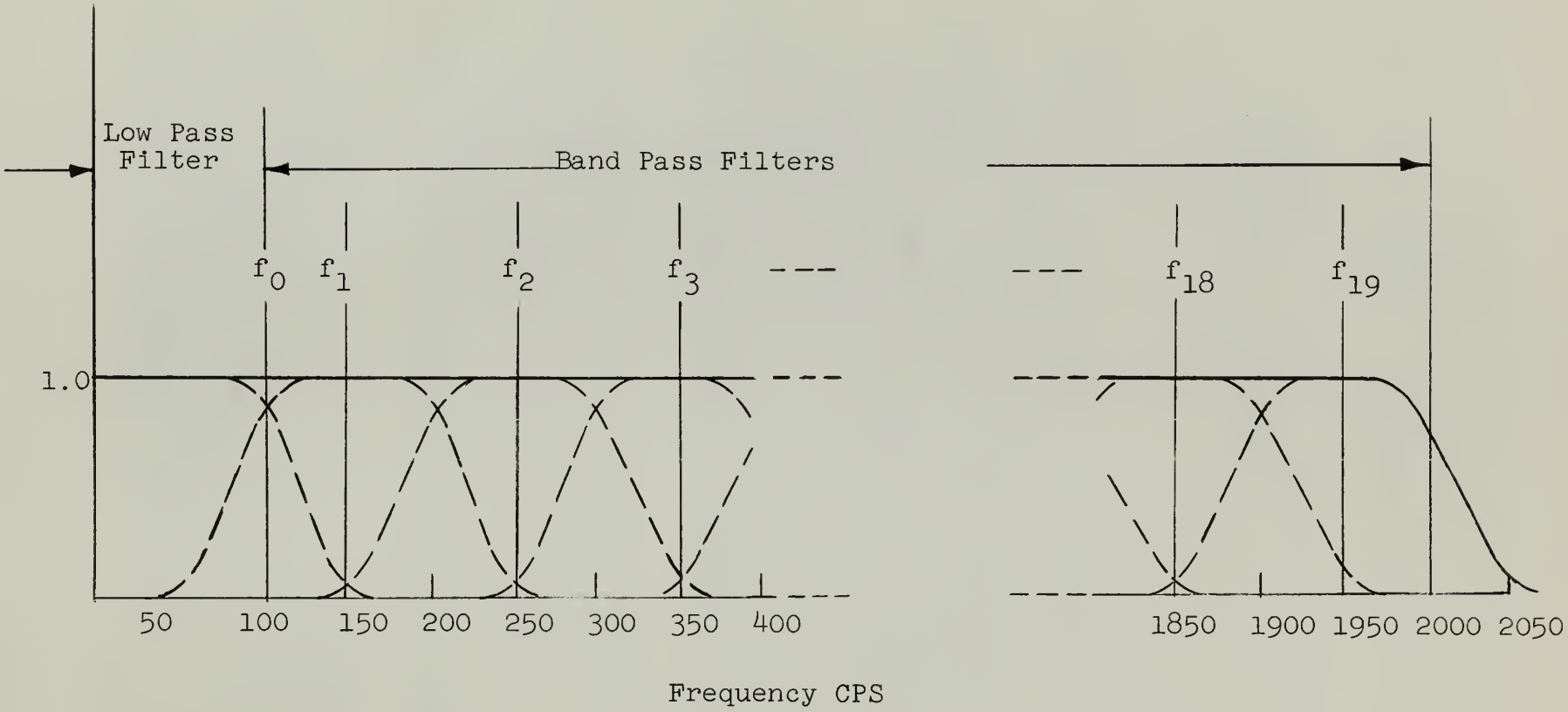


FIGURE 5-6 FREQUENCY RESPONSE OF FILTER NETWORK

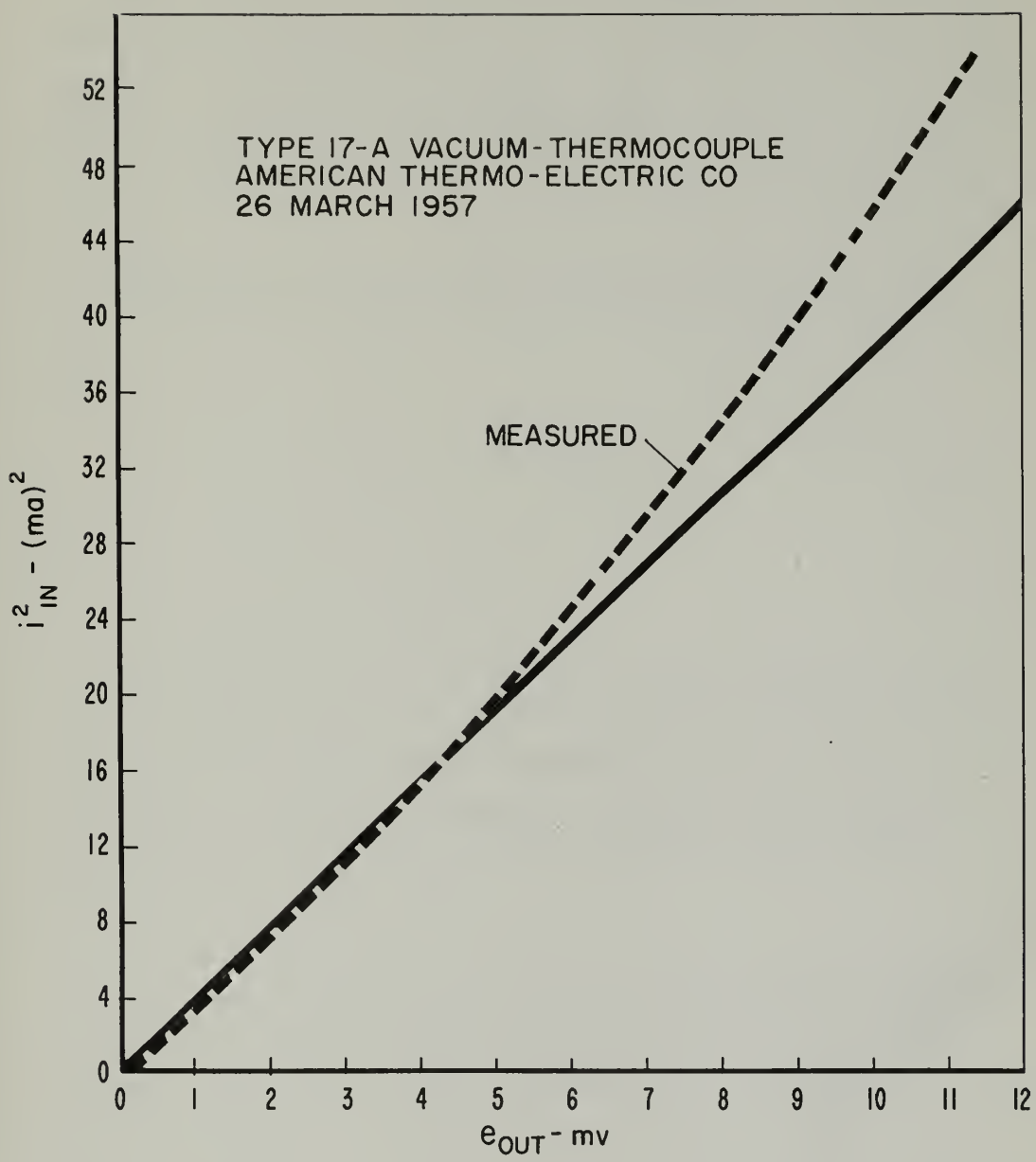


Fig. 5-7 Squared Current Input vs. Output Voltage for Thermocouple

With an alternating current input there is very little flutter perceptible in the output voltage. This is true in the type tested at frequencies approaching DC. Holding the input constant and varying the frequency from DC resulted in constant output over the range 0-2KC. The tests conducted used a type 17-A vacuum thermocouple manufactured by American Thermo-Electric Co. of Los Angeles, California.[27] Tests were conducted on 26 March 1957.

Test Results obtained with a typical vacuum thermocouple

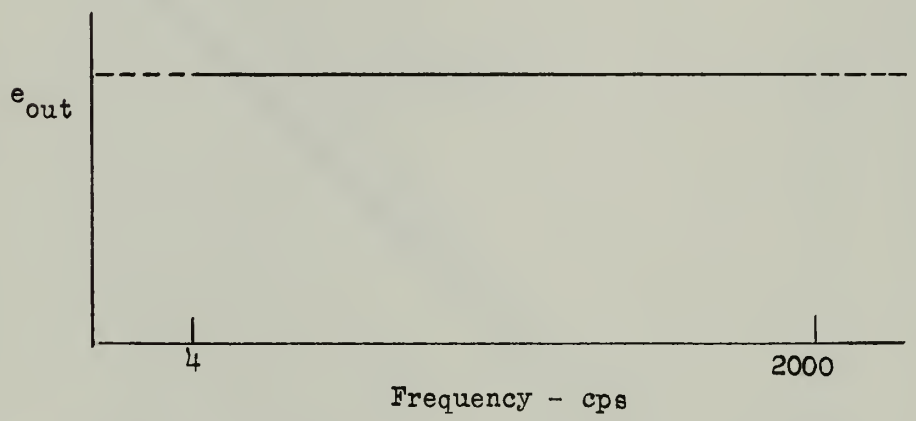


Fig. 5 - 8 Output Vs. Frequency for Thermocouple

TABLE 5-1 Squared Current Input Versus Generated Voltage

i_{in-ma}	e_{out-mv}	$i_{in}^2 - ma^2$
1.0	.4	1.0
2.0	1.2	4.0
3.0	2.5	9.0
4.0	4.2	16.0
5.0	6.0	25.0
6.0	8.24	36.0
7.0	10.5	49.0

b. Selection and Calibration

The thermocouples used in this system must be carefully selected so that limitations are not exceeded. Errors occurring when the instrument is used beyond its linear range may be unacceptable. The thermocouples used must have a rise time consistent with the frequencies applied so that the flutter is negligible. The thermocouple is used to generate a mean square approximation, as in Eq. [2-42], over a finite interval of time. The length of this interval is related to the period of flutter. Since the thermocouple is a square law device its output is proportional to the Mean Square because the thermocouple output is:

$$E_{out(TC)} \cong \frac{1}{T}\int_{o}^{T} [i(t)]^2 \, dt \qquad [5\text{-}7]$$

where T is the flutter period.

When the necessary instrumentation and calibration is made [5-7] forms the basis of generating a power density spectrum. Since i(t) is proportional to a portion of the instantaneous acceleration along the input axis, $E_{out(TC)}$ is approximately proportional to the power spectral density of this bandwidth over the time interval (dt).

Careful calibration must be made so that each thermocouple output is proportional to the square of the input voltage from its filter. The system must also be calibrated so that the summation of bandwidth thermocouple outputs is approximately equal to the output of the thermocouple whose input is the total acceleration signal. White noise or any known periodic function may be used for this calibration. Since only the thermocouples are subject to pronounced drift, they may be subsequently checked and compared at DC.

c. Recording

The thermocouple outputs must also be recorded. Since the outputs are transient in nature and low in frequency recording galvanometers are useful. There are some types available which have a flat frequency response from dc to about 100 cps.[28]

CHAPTER 6

CONCLUSIONS AND ADDITIONAL RECOMMENDATIONS

6.1 General

It is concluded that the system as presented will measure, record and analyze, in terms of its power spectral density, the accelerations experienced by missile components and equipment.

The normalized acceleration signal, which represents the component environment, can be obtained through the use of commercially available equipment in such a manner that it is able to be recorded and reproduced by commercial tape recorders. Furthermore, through the instrumentation of key equations by filters and thermocouples,this normalized acceleration signal can be analyzed and a line spectral approximation of acceleration power spectral density generated. This approximation, considering the many components involved in the system, should approach 90% of the actual input, which is a useful figure as far as missile equipment design is concerned.

However, there are some additional recommendations that might be of help in instrumenting this system.

6.2 Spectrum Analyzer

To assure that no important acceleration components are located between filter center frequencies, and therefore not weighted properly, a continuous spectrum analyzer should be used as a check.

6.3 Extension of System

To meet everchanging Military requirements and specifications it will be necessary to extend both the frequency and acceleration range. Better accelerometers are being built and it is only necessary to insert them into the system. Minor changes would have to be made in the remainder of the system but the basic ideas remain the same.

6.4 Impedance Reduction

With the advent of better accelerometers it is expected that an increase in freqency range of the low frequency accelerometer will greatly simplify the system. If the low range is increased by a magnitude to approximately 300 cps the low cutoff point of the high range accelerometer can be increased by a magnitude to 300 cps from 30 cps. It will then be possible to decrease the input impedance R_A of Fig. 3-2 by perhaps a magnitude. This will either eliminate or greatly simplify the cathode follower.

6.5 Alternate Recording Method

An alternate and perhaps better method would be to combine the high frequency acceleration signal directly with the low frequency suppressed carrier signal instead of demodulating the low frequency signal. This is an aid in recording because the low frequencies are not present as such and the recorders' task is simplfied. Some magnetic tape recorders have low frequency limitations and these types may now be used. This is especially true if the impedance reduction of Article 6.4 can be performed.

The demodulation must now take place in the analyzing system and is a simple matter. It might be done as in Fig. 6-1.

Note: 50 CPS used as Crossover Frequency,
only 0-2000 CPS of Interest

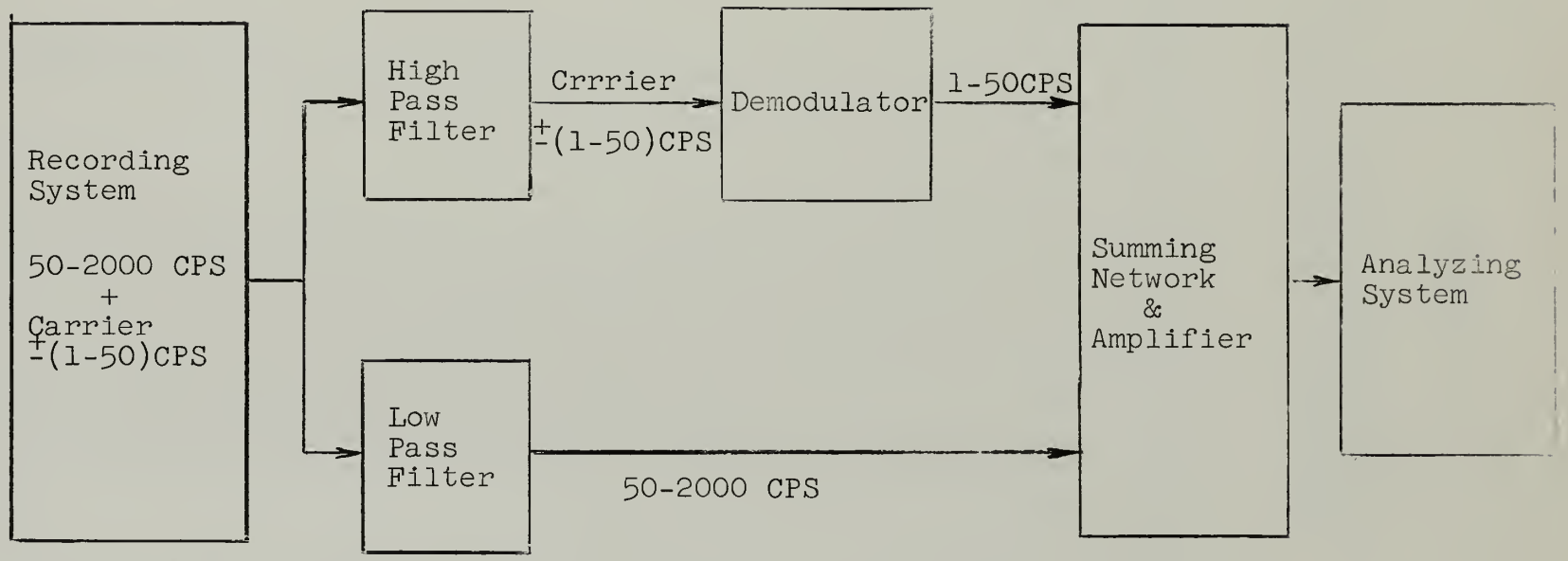


FIGURE 6-1 DEMODULATION AND FILTER CIRCUIT FOR ALTERNATE RECORDING METHOD

APPENDIX A

OUTPUT VS EXCITATION FREQUENCY DATA OF A TYPICAL LOW FREQUENCY ACCELEROMETER

A-10 Accelerometer Serial 201; Dynamic Measurements Company.
Input acceleration - 1g; Input voltage - 6V; March 26, 1957.

Excitation Frequency	Output mv/g
100	0.525
200	1.05
300	1.6
400	2.2
500	2.7
600	3.3
700	3.8
800	4.3
900	4.8
1000	5.7
1500	8.6
2000	11.5
2500	14.0
3000	17.0
3500	19.0
4000	21.0
5000	24.0
6000	26.0
7000	28.0
8000	29.0
9000	29.2
10000	29.3

APPENDIX B

FILTER DESIGN

B.1. Filter Design Specifications and Computations

$\omega_o = 2\pi f_o$ rad/sec; $f_o = 100$ cps

1 low pass filter
19 band pass filters, BW = 100 cps

Low Pass Filter

$$200\pi = \frac{1}{\sqrt{LC}}, \text{ choose } \underline{L = 10 \text{ henrys}}$$

$$C = \frac{1}{400000\ \pi^2} = .253 \times 10^{-6} \text{ farads}$$

$$C = 0.253\ \mu f$$

No. 1 Bandpass Filter

$$\omega_1 = 2\pi f_1 \quad ; \quad f_1 = 150 \text{ cps}$$

$$\omega_1 = 300\pi \text{ rad/sec}$$

$$C_1 = \frac{1}{\omega_1^2 L} = \frac{1}{(300\pi)^2(10)} = \frac{1}{900{,}000\pi^2}$$

$$C_1 = 0.1123 \times 10^{-6} = \underline{0.1123\ \mu f}$$

$$L_1 = \frac{1}{\omega_1^2 C} = \frac{1}{(300\pi)^2(.253)(10^{-6})} = \frac{10^6}{(90000)(\pi^2)(.253)}$$

$$L_1 = \underline{4.44 \text{ henrys}}$$

No. 10 Bandpass Filter

$$\omega_{10} = 2\pi f_{10} \qquad f_{10} = 1050 \text{ cps}$$

$$\omega_{10} = 2100\pi \text{ rad/sec}$$

$$C_{10} = \frac{1}{\omega_{10}^2 L} = \frac{1}{(2100)^2(\pi^2)(10)} = \frac{1}{44{,}100{,}000\pi^2}$$

$$C_{10} = \underline{0.0023\ \mu f}$$

$$L_{10} = \frac{1}{\omega_{10}^2 C} = \frac{400{,}000\pi^2}{4{,}410{,}000\pi^2}$$

$$L_{10} = \underline{0.091 \text{ henrys}}$$

No. 19 Bandpass Filter

$$\omega_{19} = 2\pi f_{19} \qquad f_{19} = 1950 \text{ cps}$$

$$\omega_{19} = 3900\pi \text{ rad/sec}$$

$$C_{19} = \frac{1}{\omega_{19}^2 L} = \frac{1}{(3900)^2(\pi^2)(10)} = \frac{1}{152,100,000\pi^2}$$

$$C_{19} = 0.000665 \ \mu f$$

$$L_{19} = \frac{1}{\omega_{19}^2 C} = \frac{1}{15,210,000\pi^2} \times \frac{400000\pi^2}{1}$$

$$L_{19} = 0.0263 \text{ henrys}$$

The remainder of the L_i and C_i values can be found in TABLE B-1.

B.2. Frequency Response Calculations

The calculations are for the circuit in Fig. 5-4.

For a fixed center frequency ω_i, the only variables are ω and ζ. The gain portion of the frequency response is presented in Fig. B-1. ζ was varied to show its effect on the response of the No. 10 Bandpass Filter.

The following contains the derivation of an expression for $\left|\frac{e_{out}}{e_{in}}\right|$ as a function of ω.

TABLE B-1

BAND PASS FILTER CONSTANTS

ω_i rad/sec	f_i cps	L_i henrys	C_i μf	Q
ω_0	100	0	0	--
ω_1	150	4.44	0.1123	1.5
ω_2	250	1.60	0.0404	2.5
ω_3	350	0.817	0.0206	3.5
ω_4	450	0.493	0.0125	4.5
ω_5	550	0.331	0.00836	5.5
ω_6	650	0.237	0.00598	6.5
ω_7	750	0.178	0.0045	7.5
ω_8	850	0.1385	0.0035	8.5
ω_9	950	0.111	0.0028	9.5
ω_{10}	1050	0.091	0.0023	10.5
ω_{11}	1150	0.0757	0.00191	11.5
ω_{12}	1250	0.0640	0.00162	12.5
ω_{13}	1350	0.0548	0.00139	13.5
ω_{14}	1450	0.0477	0.00120	14.5
ω_{15}	1550	0.0417	0.00105	15.5
ω_{16}	1650	0.0367	0.00093	16.5
ω_{17}	1750	0.0327	0.000826	17.5
ω_{18}	1850	0.0292	0.000738	18.5
ω_{19}	1950	0.0263	0.000665	19.5

$$\frac{e_o}{e_{in}} = \frac{\dfrac{\dfrac{j\omega L_i}{j\omega C}}{j\omega L_i + \dfrac{1}{j\omega C}} \quad \dfrac{(j\omega C)}{(j\omega C)}}{R + j\omega L + \dfrac{1}{j\omega C_i} + \dfrac{\dfrac{j\omega L_i}{j\omega C}}{j\omega L_i + \dfrac{1}{j\omega C}} \dfrac{(j\omega C)}{(j\omega C)}}$$

$$\frac{e_o}{e_{in}} = \frac{\dfrac{j\omega L_i}{1 - \omega^2 L_i C}}{\left[j\omega L + \dfrac{1}{j\omega C_i}\right]\left[\dfrac{j\omega C_i}{j\omega C_i}\right] + R + \dfrac{j\omega L_i}{1 - \omega^2 L_i C}}$$

$$\frac{e_o}{e_{in}} = \frac{\dfrac{j\omega L_i}{1 - \omega^2 L_i C}}{\dfrac{1 - \omega^2 LC_i}{j\omega C_i} + R + \dfrac{j\omega L_i}{1 - \omega^2 L_i C}}$$

Using $\omega_i^2 = \dfrac{1}{L_i C} = \dfrac{1}{L C_i}$

$$\frac{e_o}{e_{in}} = \frac{\dfrac{j\dfrac{\omega}{\omega_i}\ (\omega_i L_i)}{1 - \dfrac{\omega^2}{\omega_i^2}}}{\dfrac{1 - \dfrac{\omega^2}{\omega_i^2}}{j\dfrac{\omega}{\omega_i}\ (\omega_i C_i)} + R + \dfrac{j\dfrac{\omega}{\omega_i}\ (\omega_i L_i)}{1 - \dfrac{\omega^2}{\omega_i^2}}}$$

Let $\frac{\omega}{\omega_i} = X$

$$\frac{e_o}{e_{in}} = \frac{\frac{j(\omega_i L_i) X}{1 - X^2}}{\frac{1 - X^2}{j(\omega_i C_i) X} + R + \frac{j(\omega_i L_i) X}{1 - X^2}}$$

Let $\frac{X}{1 - X^2} = F$

$$\frac{e_o}{e_{in}} = \frac{j(\omega_i L_i) F}{R + j(\omega_i L_i) F + \frac{1}{j(\omega_i C_i) F}}$$

$$\frac{e_o}{e_{in}} = \frac{1}{1 - \frac{1}{\omega_i^2 L_i C_i F^2} - \frac{j R}{\omega_i L_i F}}$$

$$\left| \frac{e_o}{e_{in}} \right| = \frac{1}{\sqrt{\left[1 - \frac{L}{L_i F^2}\right]^2 + \left[\frac{R}{{}_i L_i F}\right]^2}}$$

An equivalent equation;

$$\frac{e_o}{e_{in}} = \frac{1}{\left[-\frac{\omega^2}{\omega_o^2} + \frac{2 j \zeta \omega}{\omega_o} + \frac{C}{C_i}\right]\left[1 - \frac{\omega_i^2}{\omega^2}\right] + 1}$$

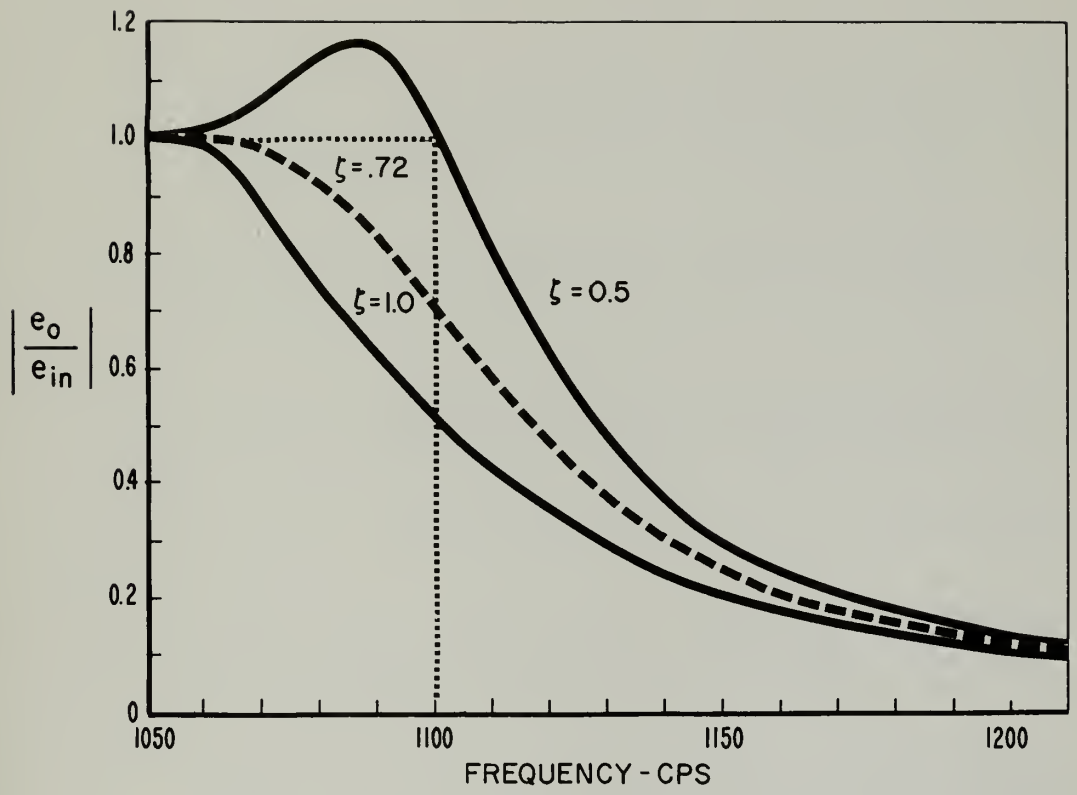


Fig. B-1 Effect of ζ on Gain vs. Frequency

APPENDIX C

BIBLIOGRAPHY

1. Morrow, C. T., How Environmental Test Conditions and Their Combination Influence the Reliability of a Product, Ramo-Wooldridge Corporation.

2. Airborne Electronic Equipment WS-107A Environmental Test Requirements, November 1955, as ammended by GMRD-E-56-1 July 1956.

3. Kaufman, J., A Re-evaluation of Vibration Testing Techniques, Electrical Manufacturing, November 1955.

4. Booth, G.B., Random Motion, Product Engineering, November 1956.

5. Laning, J. H., and Battin, R.H., Random Processes in Automatic Control, McGraw-Hill Book Company, Inc., New York, N. Y., 1956.

6. Truxal, J.G., Automatic Feedback Control System Synthesis, McGraw-Hill Book Company, Inc., New York, N.Y., 1955.

7. Seamans, R.C., Jr., Automatic Control of Aircraft, 16.40 Notes, Aeronautical Engineering Department, MIT, February 1955

8. Draper, C.S., McKay, W. and Lees, S., Instrument Engineering, Volume III, McGraw-Hill Book Company, Inc., New York, N.Y., 1955.

9. Locke, A.S., Guidance, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1955

10. Broch, J.T., Electrical Measurement of Mechanical Vibrations, Brush Technical Review, No. 1, 1957.

11. Bruel, P. V., Modern Accelerometers, Brush Technical Review, No. 2, April 1955.

12. Kert, D.B., Transducers, Allen B. Du Mont Laboratories, Inc., 1953.

13. Booth, K. P. Tape Recording for Telemetering and Data Analysis, Teletech, May and June 1952.

14.* Ampex Corporation
934 Charter Street
Redwood City, California

15.** Endevco, Series 2200 Accelerometers, 1956

16.* Endevco Corporation
161 E. California Street
Pasadena, California

17.** Glennite, Self Generating Accelerometers, Bulletin A403, Gulton Manufacturing Corporation, Metuchen, New Jersey.

18.* Gulton Industries, Inc.
212 Durham Avenue
Metuchen, New Jersey

19. Gray, T.S., Applied Electronics, John Wiley and Sons, Inc., New York, N. Y., 1955.

20.** Glennite Bulletin AVR-250, Variable Reluctance Accelerometers, December 1956.

21.** Glennite, Differential Transformer Accelerometer, Bulletin ADT - 700, November 1956.

---

* Manufacturers' Address

** Manufacturers' Bulletin

22. Morrow, C.T., and Muchmore, R.B., Shortcomings of Present Methods of Measuring and Simulating Vibration Environments, Journal of Applied Mechanics, September 1955.

23.** Northam, Model A-8, Miniature Accelerometer, 1957

24.* North American Instruments, Inc.
2420 North Lake Avenue, Altadena, California.

25. Laws, F.A., Electrical Measurements, McGraw-Hill Book Company, Inc., New York, N.Y., 1938.

26. Endevco Corporation, Instruction Manual on Model No. 2213 Accelerometer, December 1954

27.* American Thermo-Electric Company
7269 Santa Monica Blvd.
Los Angeles, California

28. Gross, E.E., Measurement of Vibration, General Radio Company, Cambridge, Mass., 1955.

29.* Test Equipment Sales Co.
29-46 Northern Blvd.
Long Island City, 1, New York

30. Diechmiller, A.C., Measurements of Shock and Vibration Under Extreme Environmental Conditions, Electrical Manufacturing, January 1957.

31. Reintjes, J.F., and Coate, G.T., Principles of Radar, McGraw-Hill Book Company, Inc., New York, N.Y., 1952.

32. Bradford, R.S., Phase-Coherent Vibration in Missiles, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, October 1956.

33. Guttwein, G.K., and Dranetz, A.I., Self-Generating Accelerometers, Electronics, October 1951.

34.** Glennite, Self-Amplifying Accelerometers, Bulletin A 500 May 1955.

JA 17 58 BINDERY

Thesis
A7257 Arnold 36042
A wide band acceleration measuring, recording and analyzing system.

JA 17 58 BINDERY

Thesis
A7257 Arnold 36042
A wide band acceleration measuring, recording and analyzing system.

thesA7257
A wide band acceleration measuring, reco
3 2768 002 01256 9
DUDLEY KNOX LIBRARY